From *SIAM News*, Volume 39, Number 4, May 2006

The Infrastructure of Mathematics: The Case of William Playfair

The Commerical and Political Atlas and Statistical Breviary. A Facsimile of the Third Edition (1801). *By William Playfair, edited and with an introduction by Howard Wainer and Ian Spence, Cambridge University Press, Cambridge, UK, 2005, 248 pages, $39.99.*

BOOK REVIEW

By Philip J. Davis

Things we now take absolutely for granted did not, at some point in history, exist: a flight from Newark to Copenhagen, the decimal system, zippers, MATLAB.

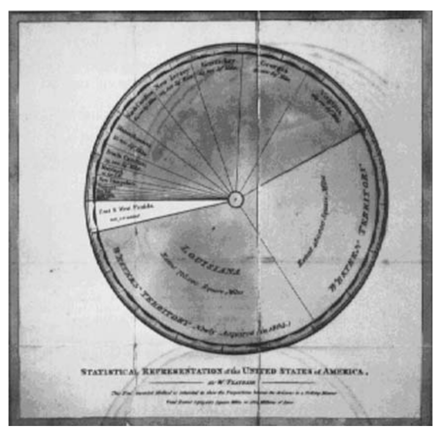
A U.S. Senator, in a televised speech from the Senate floor, uses a chart to drive home a political point. The chart displays a graph: The horizontal axis shows time; the vertical axis displays a certain quantity, perhaps money, perhaps population. The two are connected by a curve. Perhaps two such curves are displayed on the same chart. Call this a line graph. The representation is vivid. The TV audience understands what is going on. Once the Senate gets wired up for PowerPoint displays, the members will dispense with their cardboard charts and make political statements with bullets, animated graphs, and cinematic material, both real and virtual.

Line graphs are now everywhere. An understanding of how they present variations is assumed to be a part of elementary-school education. The National Numeracy Strategy sets as a goal:

> "In science pupils will, for example, order numbers, including decimals, calculate simple means and percentages, use negative numbers when taking temperatures, decide whether it is more appropriate to use a line graph or bar chart, and plot, interpret and predict from graphs."

Inexpensive hand-held graphing calculators stand ready to help the students represent data graphically, or switch back and forth between tabular values and symbolic expressions.

It was not always thus. The first person to make use of line graphs, bar graphs, pie charts, and color coding in a substantial way to represent social and economic data was William Playfair (1759–1823). Previously, such numbers were displayed in columns. Playfair missed out on scatter diagrams—well, no one can think of everything. The book under review, a facsimile edition of two of William Playfair's works, along with an introduction by Ian Spence, a professor of psychology at the University of Toronto, and Howard Wainer, a member of the National Board of Medical Examiners in the U.S. Both men are aficionados and historians of graphical representations of data.



William Playfair's 1805 pie chart showing the areas of states in the USA. Little Rhody's slice is hardly visible; the Louisiana and Western Territories loom large.

In *The Commercial and Political Atlas* Playfair charts the balance of trade between England and 23 other countries. The chart for trade with North America, which covers the period of the American Revolution, itself makes the book worth a look. Also charted are financial statistics, such as the English national debt and expenditures of the army. The *Statistical Breviary* provides a set of statistics for several countries, mostly European, such as size, population, size of navy (in 1800, Portugal had 18 ships, each with 40 to 80 guns), with visual comparisons presented via circular discs of various sizes.

William Playfair was a colorful, nonconformist, jack-of-all-trades Scot. A skilled engraver, he produced some of James Watt's engineering drawings; he was also a not very successful technological entrepreneur and speculator, a disreputable envelope-pusher who was charged with embezzlement, a prolific author whose books range from *The History of Jacobinism* to *Parliamentary Reform* to *European Trade*. Playfair fell out with the Edinburgh establishment after publishing negative remarks about the economist Adam Smith. He lived for some time in revolutionary France but then fell out with the revolutionaries. There is no authenticated portrait of Playfair, but Google will provide you with a portrait of a man, wig and all, designated as William Playfair, which co-editor Ian Spence assures me is doubtful. (Watch out! The computer is not the absolute arbiter of truth!)

I had never heard of William Playfair, and when I read that he was essentially the first to display socioeconomic data in line-graph format, I was incredulous. And I found it equally incredible that it was not until the middle of the 19th century that Playfair's idea was picked up seriously. W.S. Jevons, the logician and economist, advocated its use. Biometrician Karl Pearson used it. Florence Nightingale (the first woman to become a member of the Royal Statistical Society) produced a pie chart to depict the outcome of patients in her military hospital.

Plugging his method, Playfair wrote in the *Breviary*,

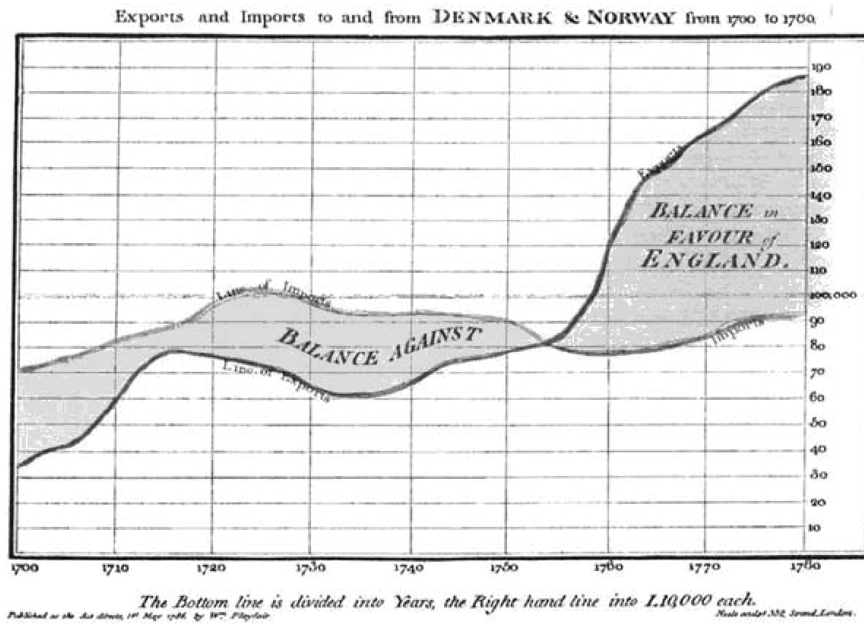
> "The advantages proposed by the [graphical] mode of representation are to facilitate the attainment of information, and aid the memory in retaining it: which two points form the principal business we call learning. . . . of all the senses, the eye gives the liveliest and most sensible idea of whatever is susceptible of being represented to it. . . ."

Playfair also found it necessary to walk his readers through interpretations of his line graphs:

"Suppose you want the amount of exports in the year 1750. Observe where the line of exports passes the line marked at the bottom 1750, and by looking at the right hand margin, you will find it 13,300,000."

Incredible. Well, I can't understand most of what a contemporary user's guide tells me to do; I need a maven to walk me through.

In view of the manifest advantages, why was the line graph so slow in arriving? Indications of the existence of the line graph go back as far as the 10th century, but the idea never took off. The editors of the Playfair facsimile suggest a number of reasons: The difficulty of engraving copper plates. Loss of accuracy, the possibility of misrepresentation, mistrust of the empirical and the visual on the part of the rationalists who asserted that knowledge is derived from reason. And even this: doubt as to whether a stretch of time or an amount of money could be represented by a length! This is a mixture of modes that Aristotle found unacceptable, calling it "metabasis." In a more general context, I have developed these reasons in an essay called "The Decline and Resurgence of the Visual in Mathematics" (as yet unpublished).



Balance of trade 1700–1780: England vs. Denmark and Norway. From The Commercial and Political Atlas and Statistical Breviary.

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If you look for William Playfair's name in standard histories of mathematics, say in Kline, in Grattan-Guinness, or in Boyer and Merzbach, you will look in vain. You may find the name Playfair in their indices, but the reference will be to John Playfair (1746–1819), William's older brother, a mathematician mildly famous for recasting Euclid's notorious Fifth Axiom in the form in which we now most often cite it—Playfair's Axiom: *Given a line and a point not on the line, it is possible to draw exactly one line through the given point parallel to the line.*

Here at Brown University, I have been participating in an ongoing discussion group on the philosophy of mathematics. Our text (for fall 2005) was the Introduction to Carlo Cellucci's *Filosofia e matematica* (an English translation is as yet available only for the Introduction). In it, Cellucci lists thirteen "dominant views" about mathematics and then proceeds to demolish them all. Sixth on the list is: "According to the dominant view, mathematics is theorem proving because it 'is a collection of proofs.' " In *Alice in Wonderland*, Humpty Dumpty inferred that everyone is free to make his/her own definitions. If the dominant view delimits mathematics to theorems, let the dominators feel free to do so. I personally allow a much wider definition of what constitutes mathematics.

Without going into the question of who it is exactly that espouses the various dominant views—perhaps Cellucci has simply set up straw men to knock down—I would say that the neglect of Playfair in the history of mathematics fits right into what Cellucci is saying. Line graphs, bar graphs, pie charts are not in themselves theorems. They are ways of presenting mathematical material so as to make the material more comprehensible and even to elicit inferences and further developments. They are part of what might be called the "infrastructure of mathematics."

This phrase is rarely used in discussions of mathematics and carries a variety of meanings when it is. Some use it to refer to mathematical education. Fields medalist William Thurston has spoken of the "basic mental infrastructure of mathematics." To me, the infrastructure of mathematics includes such felicitous notations as x^2, parentheses ( ), perhaps even the ugly blob of black ink ■ or the more aesthetic □ that displaced the classic Q.E.D. (Was it Paul Halmos who introduced the latter?) It includes MATLAB and all computer packages and digitized online versions of the mathematical corpus. Following the lead of the late algebraic number theorist Serge Lang, who called mathematical axioms "the hygiene of mathematics" (*The Beauty of Doing Mathematics: Three Public Dialogues*, Springer-Verlag, 1985), and thinking of hygienic urban infrastructures, I might even include axioms as part of the mathematical infrastructure.

Histories of mathematics are concerned largely with the presentation and development of theorematic material; infrastructure gets short shrift. After all, the most obvious portions of the infrastructure are the common languages, such as Greek, English, Danish, in which mathematics lies embedded, and what need is there to mention this fact? Yet without an infrastructure, there would be no mathematical enterprise. For the time being, I would say that the infrastructure of mathematics consists roughly of the accepted devices used in the communication and the carrying through of certain parts of the implied inferential structure of the subject. I am not yet ready to provide a sharper definition.

I applaud the appearance of this facsimile edition of Playfair. The authors claim for Playfair's line graphs and other graphical devices a revolutionary Kuhnian change of paradigm. This may be a bit much. But this claim serves to suggest to me that, in the long run, the computer together with computer graphics and other mathematical creations and devices that I have now relegated to the notion of "infrastructure" may very well create a revolution in mathematical methodology and content. They may even provide the *coup de grace* to the view that mathematics is limited to theorem proving.

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