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Beyond the Pillars of Hercules: Soft Mathematics

Goodbye, Descartes: The End of Logic and the Search for a New Cosmology of the Mind. *By Keith Devlin, John Wiley, New York, 1997, 320 pages, $27.95.*

(Also considered in this review: "Devlin's Angle: Soft Mathematics," Keith Devlin, www.maa.org/devlin/devlinangle_april.html; and "Ten Remarks on Husserl and Phenomenology," Gian-Carlo Rota, address delivered at the Provost's Seminar, Massachusetts Institute of Technology, 1998.)

BOOK REVIEW

By Philip J. Davis

Years ago I used to hear a plaintive wail that the "soft sciences"—sociology, psychology, anthropology—were still awaiting the arrival of their Newton, someone to do for them what Newton's mathematics has been able to do for physics. On hearing this wail, some would reply: "Newton? Why, they are still awaiting their Galilean revolution!"

The desire for a more precise, more mathematized sociology, psychology, and so forth, is still with us. And whether the desire and the push come principally from the sociologists themselves, or from members of the applied mathematics community looking for new worlds to conquer, I cannot tell.

The current "wail" is slightly different: I don't hear much about a new Newton (as an individual genius), but I hear a fair amount about the possibilities of a "new mathematics" that, it is hoped, will do the trick. What trick? Clearing up the mess in Bosnia?

Keith Devlin is a college administrator, a mathematician, a logician, a promoter of "business applications of situation theory," and a prolific popularizer. In his book, which is largely about the difficulties encountered in the formalization of natural languages, he presents an exposition and a history of such attempts up through Chomsky and the post-Chomsky theorists. The book is stimulating to the point of annoyance, and I choose to exercise my reviewer's privilege and skip most of his arguments, building my article around his last chapter, "Soft Mathematics."

Devlin points out, correctly, that the "hard" sciences are locked into a framework of thought, action, and aspiration laid out by René Descartes. Features of the Cartesian mode include observation as opposed to tradition or revelation, reduction of the whole to constituent parts, the separation of mind and matter. Objections to and limitations of the Cartesian mode have been trumpeted ever since Descartes put it on our plate, and even as its successes have overwhelmed us.

Listen to the words of Denis Diderot (1713–1784), a man of the French Enlightenment and the principal Encyclopedist, who was reasonably knowledgeable about the mathematics of his day. Mathematics is grand, said Diderot, but it is played out and exhausted as far as utility is concerned. If mathematics were so all-revealing,

> "what purpose would all those deep theories of celestial bodies and all those enormous calculations of rational astronomy serve, if astronomy cannot dispense with [James] Bradley and [Pierre Charles] Le Monnier observing the heavens?"

Mathematics has reached the limits of its power, according to Diderot:

> "This science will come to an end suddenly where the Bernoullis, Euler, Maupertuis, Clairaut, Fontaine [Alexis Fontaine de Bertins (1704–1771), early developer of the calculus along with these others] and d'Alembert have left it. It has set up Pillars of Hercules beyond which there is no going. In the centuries to come, their work will persist even as the Egyptian pyramids with their masses full of hieroglyphs awaken in us the awesome idea of the power and resources of the men who raised them."

Just what mathematics can produce before we have to throw in the towel is by no means clear, particularly since a good deal of socially relevant mathematics is put in place by fiat or prescription.

In this century, the limitations perceived as a consequence of Gödel's theorem, or the physical limitations on the speed of computation or the limitations due to chaos or the butterfly phenomenon, all seem to present stone walls of impossibilities. They assert the powerlessness of mathematics beyond a certain point, although some observers have said that the argument has holes in it because of the gap that exists between abstract and physical counting. The moral of Devlin's book, as indicated by its title, is that if we are to make further progress, it is time to kick Descartes and his method out the window.

We are now in the presence of mathematical power far beyond the material familiar to Diderot, and, indeed, there are those in each generation, including ours, who have said: It's all finished; we have only to dot the i's and cross the t's. In point of fact, though, just what mathematics can produce before we have to throw in the towel is by no means clear, particularly since a good deal of socially relevant mathematics is put in place by fiat or prescription.

Devlin throws in the towel as regards the "soft sciences":

> "Cartesian science and mathematics have taken us a long way in our understanding of the physical world. They have also provided us with

significant—though hitherto far less useful—insights into the world inside our minds. But there is growing evidence that, when it comes to the human and cognitive sciences, we are reaching the limits of the understanding that can be achieved through the traditional tools and methods of science and mathematics."

But not quite. In Devlin's view, mathematics will not be totally dethroned even as our understanding of the human and cognitive sciences is developed by other means:

"The new sciences will involve a mixture of mathematical reasoning and the less mathematically formal kinds of reasoning used in the social sciences. A reasonable name for such reasoning would be soft mathematics."

As regards the new soft mathematics, Devlin predicts that "Many scientists and mathematicians, trained in the traditions of their fields, will throw up their hands in horror upon hearing someone claim that we need to look for ways of understanding that go beyond the traditional methods."

Just what is "soft mathematics"? It doesn't yet exist, so how can Devlin really say? Is it soft by virtue of its subject matter, e.g., models of cognition? By the structure of the mathematics employed, e.g., definition/proof (hard) vs. intuition, diagrams (soft)? By the way the mathematics is applied to the domain, e.g., the use of computer algorithms to pick up messages hidden in the Bible?

Can anyone say, as Adams and Leverrier did with the then undiscovered planet Neptune: Focus your telescope onto a certain part of the intellectual sky; there is where you will find soft mathematics? Devlin conjectures that "situational logic," developed by Jon Barwise (and described hastily in Chapter 8), may provide a slight hint as to its location.

Seeking a "character witness" for the existence of soft mathematics, Devlin brings forward the words of mathematician/ philosopher Gian-Carlo Rota:

"Shocking as it may be to a conservative logician, the day will come when currently vague concepts such as motivation and purpose will be made formal and accepted as constituents of a revamped logic, where they will at last be allotted the equal status they deserve, side-by-side with axioms and theorems."

While Devlin seems to have come to his belief through the difficulty of dealing logically and computationally with the ambiguities of natural language, Rota came to it through his admiration for the writings of mathematician/philosopher Edmund Husserl (1859–1938), the father of phenomenology. I agree with Devlin when he says that, judging from his hope for formalizations, Rota does not appear ready to defenestrate Descartes.

In a later statement, Rota, interpreting Husserl for us, says that what characterizes mathematics is "(1) absolute truth (2) items, not objects (3) non-existence (4) identity (5) placelessness (6) novelty and (7) rigor."

I pass these characteristics on here without further explanation or comment. Moreover, Rota continues, if "new rigorous theoretical sciences come into being" by new Galilean revolutions, they will be characterized by the aforesaid features and by "(a) The denial of common sense and (b) Theoretical laws."

Other anticipators of soft mathematics—including mathematician/semioticist Brian Rotman—have provided different descriptions of how a new type of mathematics might arise: in the marriage of mathematics and technology with the human senses of touch and movement; in other words, in what has come to be called "virtual reality."

Let me now speculate on the eyebrow-raising, "throwing up of the hands in horror" quality of soft mathematics, which can be related to Rota's "denial of common sense." Eyebrows have been raised so often that one is apt to take as a provisional definition of soft mathematics "anything that raises the eyebrows of the establishment." The first case of eyebrow-raising I am aware of is that of the School of Pythagoras. Legend has it that when they discovered the irrationality of $\sqrt{2}$, they first raised their eyebrows and then slaughtered a hecatomb of cattle. Since then, $\sqrt{2}$, $\sqrt{-1}$, non-Euclidean geometry, Cantor's various infinities, Dirac's delta function, and numerous other theories—all eyebrow-raisers in their day—have been co-opted into establishment mathematics. Considering also that one critic called Hilbert's proof of the basis theorem for algebraic forms (1888) theology and not mathematics, it's clear that my provisional definition of soft math needs some tinkering.

More to the point here is the case of Aristotle. In his splendid book *Theology and the Scientific Imagination*, Amos Funkenstein wrote,

"The Aristotelian tradition viewed a mathematical science of change not only as imprecise and equivocal but as a downright category mistake. Mathematical objects are abstracted from all physical properties, and physics is, first and foremost, the knowledge of causes of change."

Such a mixture was called "metabasis." Later on, Archimedes mixed two types of material, mathematics (hard?) and physical experiment (soft?), to arrive at mathematical results, and he worried about it. The ancient horror of metabasis is well explained in Funkenstein's book. This horror persists today in certain quarters—in the horror of computer proofs, for example, or in the reluctance of departments to hire people whose interests involve unorthodox spans of material. The proscription of metabasis can be thought of anthropologically as a kind of purity rite.

Without formulating a personal definition of soft mathematics, I would like to assert that over the centuries, many soft forms have been around. The ones I know all involve mixing. As examples: astrology; the hermetic geometry of John Dee (1527–1608); the application of mathematics to ethics and the law by Leibnitz and later by Nicolaus Bernoulli; the mathematics-like squiggles of the school of the psychiatrist Wilhelm Reich (1897–1957), of Orgone Box notoriety, reminiscent visually of the squiggles in Frege's *Grundgesetze der Arithmetic*. I would even place in the category of soft mathematics graphic art that derives inspiration from mathematics (such as some of the productions of Marcel Duchamp (1887–1968), who is said to have read Poincaré). Classical applications of mathematics are constantly judged and filtered for purity and acceptability, so I have eliminated them from my list.

Have these forms been successful? It depends on what one means by “successful”; they certainly have affected and continue to affect people. Some even have led to significant hard mathematics.

The issues involved here are far wider and deeper, far more important than I have been able to treat in my allotted space. Ultimately, they strike at the heart of the question of whether any formalization of creativity is worth a nickel.

In summary, there has always been soft mathematics. There will always be soft mathematics. It is very difficult to predict in advance what going beyond the “Pillars of Hercules” will accomplish and just what, if anything, it will do to people or for people. We will know, in retrospect, as it plays out.

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