from *SIAM News*, Volume 34, Number 9

# The Joy and Suffering of Research

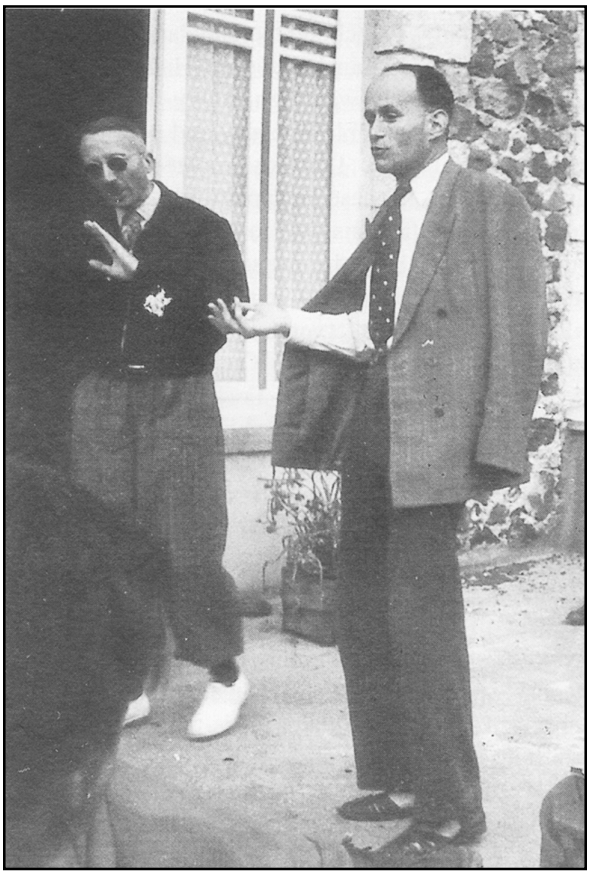
**Laurent Schwartz: A Mathematician Grappling with his Century.** *By Laurent Schwartz (translated by Leila Schneps from the 1997* **Un mathématicien aux prises avec le siècle***), Birkhäuser, Boston, 2001, 490 pages, no index (regrettably), $44.95.*

BOOK REVIEW

*By Philip J. Davis*

In August 1950, the 11th International Congress of Mathematicians met at Harvard University in Cambridge, Massachusetts. The four-year sequence of these congresses had been interrupted by World War II; the 1950 congress was the first one held since 1936. The attendees were from many parts of the world. Some areas still lay in ruins. Food supplies and heat were limited, and there was scant money for new clothes, let alone trips abroad. For the honor of an ancient discipline that for long centuries had proclaimed its universality, but that in the bitter years from 1933 to 1946 had seen this proclamation violated, mathematicians from former enemy countries faced one another and did the best they could to put aside their bitter memories.

On the 30th of the month, there was a great convocation in Sanders Theater for the presentation of the coveted Fields Medals. The theater was full. Jacques Hadamard, a short, gaunt man of eighty-five, the honorary president of the congress, walked in to a standing ovation. He was there to see Laurent Schwartz, his great-nephew (by marriage), honored. I knew from inside information that Hadamard had crossed the waters also in the hope (perhaps vain) of seeing our southwestern cacti in bloom.



*Laurent Schwartz (right) with Henri Cartan. (From* A Mathematician Grappling with His Century.*)*

On this occasion, I was in the audience in Sanders Theater, sitting below the rafters, so to speak, with a new PhD in my pocket, concerned less about what was happening on the stage than about where I could find a job. As Bertholdt Brecht wrote, "Erst kommt das Brot, dann die Kunst." (Bread comes first, then art.)

On this occasion, then, Laurent Schwartz from France and Atle Selberg from Norway were awarded Fields Medals, the former for his seminal work on distribution theory and the latter for a complex variable-free proof of the prime number theorem. Harald Bohr of Denmark was the presenter. For both of these brilliant mathematicians, the Fields was only one of the many honors they would accumulate over the years.

Schwartz was justly proud of his youthful work on the theory of distributions, which has become a standard chapter of functional analysis and a link between real analysis, linear differential equations, and probability theory. In light of this and numerous other mathematical accomplishments, the mathematical world provided him with invitations galore and red-carpet treatment.

During his professional career, Schwartz, born in Paris in 1915, the son of a distinguished surgeon, was a mathematician of the first rank, a prolific expositor, and an inspiring teacher. He was a professor at the University of Nancy (1945–52) and at the Ecole Polytechnique (1953–83). In 1938, he married Marie-Hélène Lévy, the daughter of the famous probabilist Paul Lévy. Despite an early (pre-antibiotic era) bout with TB and the birth of two children, Marie-Hélène became a mathematician in her own right, with a distinguished publishing and teaching career.

Like Vladimir Nabokov, Schwartz was a butterfly aficionado, and like many of the young intellectuals of his day he was an intense left-wing political activist (a Trotskyite from 1935 to 1947). What I didn't know as I sat in Sanders Theater, and just now learned in the book under review, is that Schwartz obtained an entry visa to the USA only after the personal intervention of President Truman (1950 being the onset of the McCarthy-era paranoia).

There is often a gap between an author's written words and his real-life personality. For all I know, not having met the man personally, he may be a genial fellow, but I've not been able to warm up to the author of this autobiography. I marvel at his mathematical brilliance; I shudder when I read of the difficulties and terrors of his miraculous survival in a France overrun by Nazi murderers, and I can hardly force myself to read the litany of his deportations.

My quick and unscientific sampling of the 490 pages of the autobiography yields: 50% devoted to politics, 25% to mathematics and mathematicians, 25% to family, people, and so forth. For all that I am pro-human rights, I am not magnetized by the minutiae of seven decades of French politics. The details of Schwartz's involvement with the Spanish Civil War, World War II, the wars in Algeria, Vietnam (both French and American), and Afghanistan, the "Committee of Mathematicians," and the squabbles (and much worse) that accompanied left and right internecine warfare were not "page-turners" for me. Yet it would be a serious

misrepresentation of Schwartz's life to fail to point to that 50%. His book is an important source of materials for historians of 20th-century left-wing European politics.

The mathematical core of the autobiography is contained in the 46-page chapter entitled "The Invention of Distributions." Now, what is a distribution in the Schwartzian sense? A distribution is a continuous linear functional on the space of infinitely differentiable functions on $(-\infty, \infty)$ with compact support. This definition, expressed in the unfortunate style of today's advanced lectures, may mean very little to the uninitiated and may intimidate them. Its significance opens up a bit when one points out that its purpose is to provide a way of legitimizing and extending the "illegitimate" Dirac delta function. Schwartz arrived at this illumination slowly—the rocky path is detailed in this chapter—and finalized it in isolated, unliberated Grenoble in February 1945. (V-E Day was May 8, 1945.) The identification of the date is in the spirit of Descartes's assertion that his revelation occurred on November 10, 1619, in Ulm.

Mathematics has often turned impossibilities into possibilities. "Four from three you cannot take"; "you cannot extract the square root of minus one"; "a singular matrix or a nonsquare matrix cannot be inverted"; "the delta function cannot exist." All these and many more mathematical no-no's have been converted into yes-yes's by a process of context extension. As regards the delta function: Inquire not about its individual values, but about its effects under convolution. Within Schwartz's theory, the delta function metamorphosed into the delta functional. Given the long record of turnarounds, I wonder in what area the next fruitful legitimization will occur.

There is a bit of a paradox with regard to distribution theory. Schwartz was influenced significantly by the newly formed (1935) Bourbaki association, but was not uncritical of the movement, particularly its abhorrence of applied mathematics. Though distribution theory has opened up new areas of mathematical research and though it has contributed a point of view, a useful attitude and mode of expression, it has done little for the actual computation required in bottom-line applied mathematics. By way of analogy, its role can be likened to that of the real number system with respect to digital computation, which cannot fully accommodate the real numbers. It's been said (jocularly) that "Mathematicians and physicists used to live in productive sin with the delta function and its derivatives. Schwartz provided them with a marriage certificate, which they take out of the closet occasionally to consult and show around."

The delta function and its related ideas had a long and fruitful pre-Schwartzian history. Schwartz himself alludes to the work of Peano, Hadamard, Heaviside, Wiener, Carson, van der Pol, Bochner, Leray, and Sobolev, among others. In his "prehistory," Jesper Lützen,* a historian of mathematics at the University of Copenhagen, takes us through many meticulously referenced steps that detail the work of all the individuals on whose shoulders Schwartz's work rested. Starting with d'Alembert and Euler (c.1750) on the vibration of strings, Lützen continues the story beyond Schwartz, describing alternative and post-Schwartzian definitions of generalized functions.

For those for whom it makes a difference, Lützen's "Concluding Remarks" and his "Notes" discuss questions of priority and the early and mixed assessments (which I believe are still valid) of the value to mathematics and physics of the theory of distributions.

Laurent Schwartz's autobiography, written in his 80th year, displays a man who cannot tell us enough about himself. His memory not only speaks, it gushes. The dense text is studded with numerous passages that run from the narrative to the speculative, from the didactic to the polemic, from the amusing to the tragic, from the profound to the trivial. He writes that

> "The alternation of joy and suffering is at the heart of research. Young people need to get used to it. High school students often think that it isn't normal to think about a problem for more than an hour."

Here I know what he's talking about.

I sympathize with the tragedies of his personal life, and know that this alternation is easily extended in his case to the whole of life. Honest with regard to the nature of creativity, Schwartz has described the blocks and declining powers of inventive abilities as the years rolled on. He is absolutely passionate both for mathematics and for politics; at the end, his political activism dominates.

The final words of his autobiography are:

> "Mathematicians transport their rigorous reasoning into situations of daily life. Mathematical discovery is subversive and always ready to overthrow taboos, and it depends very little on established powers."

Despite some ambiguity that I perceive here, I would like to think that Schwartz is saying that the mathematical spirit operating within society may of itself be a source of policies and attitudes that alternate between joy and suffering.

*Philip J. Davis, professor emeritus of applied mathematics at Brown University, is an independent writer, scholar, and lecturer. He lives in Providence, Rhode Island, and can be reached at philip_davis@brown.edu.*

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*Jesper Lützen, *The Prehistory of the Theory of Distributions,* Springer-Verlag, 1982.