

From Spots and Dots to Deep Stuff

The Geometry of Numbers. By C.D. Olds, Anneli Lax, and Giuliana Davidoff, *The Anneli Lax New Mathematical Library*, Mathematical Association of America, Washington, DC, 2000, 174 + xvi pages, \$24.95.

Although I don't normally review books whose content is largely theorematic, I make an exception in this case because of a debt of gratitude to the late Anneli Lax; during her lifetime, I never expressed this gratitude to her adequately.

Here's the story. In the late 50s and early 60s, while there were a good many interesting mathematical topics that could be made accessible to bright and interested high school students, research mathematicians were reluctant to spend time putting together such material for books. There was even a thought that such activity would result in a loss of professional status. At least that was my perception; I think it was also Anneli Lax's perception, and she determined to do something about it.

BOOK REVIEW

By Philip J. Davis

There were notable exceptions, of course: Hobgen's *Mathematics for the Millions*, Kasner and Newman's *Mathematics and the Imagination*, to name a couple of very successful titles. Going back a generation and digging deeper mathematically were Felix Klein's *Elementary Mathematics from a Higher Standpoint* (1908) and Hilbert/Cohn-Vossen's *Anschauliche Geometrie (Geometry and the Imagination, 1932)*. Poincaré of course wrote popular books, as did Courant and Robbins.

Around 1960 Anneli Lax approached me about contributing a volume for a series called The New Mathematical Library (NML) that she was editing and that was designed to overcome this reluctance. The New Mathematical Library at that time was published jointly by Random House and Yale University, and with the blessing and encouragement of the SMSG (School Mathematics Study Group), one of the early "new math" groups. I agreed and wrote *The Lore of Large Numbers*, Number 6 in the series, still in print but now horribly out of date. It was the very first of a number of books I've written at a variety of levels, and it was my introduction to the fascinating world of writing and publishing, both technical and trade. Anneli introduced me to Jason Epstein, then a young publishing whiz, who went on to make a great reputation as an executive of Random House, co-founder of *The New York Review of Books*, and as a writer.

In the early numbers of the series, the preamble to the reader states that:

"This book is one of a series written by professional mathematicians in order to make some important mathematical ideas interesting and understandable to a large audience of high school students and laymen."

Anneli succeeded admirably. Her series now contains more than forty titles, authored by some of the finest names in mathematics. As time went on, it was inevitable that the target be expanded beyond high school students:

"The NML matured into a steadily growing series . . . of interest not only to the originally intended audience, but to college students and teachers at all levels."

The Geometry of Numbers is an example of this expanded approach. The provenance of the book is of some interest: The text is derived essentially from a raw manuscript left incomplete by C.D. Olds (1912–1979), who was a professor of mathematics at San Jose State University. Anneli Lax rewrote and fine-tuned the manuscript. When she became ill, her work on the manuscript ceased, and the MAA began a search to find someone to complete the job. At this point Giuliana Davidoff, a professor at Mount Holyoke College who brought to the project her experience as a member of the Carus Monograph editorial board, stepped in. She polished the final manuscript and saw it through to publication. I'm sure that Anneli would have been very pleased with this fine introduction to the geometry of numbers.



My own first encounter with the geometry of numbers occurred when I was a sophomore in college. As a math major and habitué of the college math library, I found the recently appeared *Development of the Minkowski Geometry of Numbers* (1939) by Harris Hancock on the shelf of new books. This was also my introduction to the name of the famous Hermann Minkowski (1864–1909). Hancock (1867–1944), a professor of mathematics at the University of Cincinnati, wrote on a variety of topics in analysis. Hancock's book, although jam-packed with solid stuff, is more than a bit cluttered in its exposition, and perhaps that's why it's

not in the bibliography of the book under review.

What is the “geometry of numbers”? As developed by Minkowski, it appears to be about integral lattice points Λ in the plane (or in n -space)—I think of them as spots and dots—and their relation to sets in the plane, particularly to convex sets. From there, one goes to applications in pure mathematics, e.g., approximation of irrational numbers by rationals, representations of integers by sums of squares, problems of optimal packing. And from there, one also can go to real-world applications (mentioned but not pursued), such as advances in crystallography, superstring theory, error-detecting and -correcting codes, and data compression.

Many of the problems predate Minkowski. The representation of an integer as the sum of two squares, for example, is a question that goes back to Diophantus (c. 250). Gauss studied the number of lattice points in a circle of radius r as $r \rightarrow \infty$.

But Minkowski, and later Hans Frederik Blichfeldt (1873–1945), who was a professor at Stanford and whose work is featured prominently in the text, developed new and striking methods. These advances gave new zip to the old questions and allowed the inquiries to fan out marvelously.

As Peter Gruber, co-editor with Jörg Wills of the comprehensive *Handbook of Convex Geometry* (North-Holland, 1993), informed me,

“In classical geometry of numbers, the main aim was to prove results from diophantine approximation, algebraic number theory and theory of positive quadratic forms, by means of geometric tools. While this is of some interest, the geometric objects (lattices, reduced bases, convex bodies, lattice polytopes) are investigated mainly per se. Relations to coding, modular functions and computational geometry are also treated.”

The Geometry of Numbers limits itself to the classical portions (in Gruber’s sense) and has as its foundation stones the following two theorems:

Minkowski’s Fundamental Theorem:

Let C be a two-dimensional M -set with center at the origin O and area greater than or equal to 4. Then C contains, either in its interior or on its boundary, lattice points of Λ other than O .

(An M -set is convex and symmetric with respect to O .)

Blichfeldt’s Theorem:

If the area of a two-dimensional set C is greater than the integer n , then, by a parallel displacement, C can be made to cover at least $n + 1$ lattice points of Λ .

The book includes problems, solutions, historical notes, and bibliographical references that go beyond undergraduate enrichment.

The perception that popular or expository writing is a no-no for the research mathematician is still around. There are always exceptions, of course. Greater flexibility might be inculcated in the graduate years, but, alas, it is not. Even SIAM has suffered from this narrow, self-limiting sentiment. I. Edward Block, founder and managing director of SIAM for many years, started up *SIAM Review* in 1959; he told me recently that at the beginning, getting genuine review articles for this journal was worse than pulling teeth.

The Anneli Lax New Mathematical Library is a continuing legacy and an excellent medium for those mathematicians who wish to speak to more than just the members of their special-interest groups.

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