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Exciting Times for Probability Research

By Philip Protter and Ed Waymire

A workshop titled Current and Emerging Opportunities in Probability was held at the National Science Foundation, May 29–31, 2002. The workshop was organized by Philip Protter, Cornell University, and Ed Waymire, Oregon State University, with the objective of identifying future directions and opportunities for probability research. The 18 participants were probabilists from universities across the U.S. and Canada.

Workshops of this type are not unusual at NSF, although this one may have been the first to be sponsored solely by the probability program. A separate workshop had been held for NSF's statistics program a few weeks earlier. As the organizers, we took something of a "blank slate" approach in setting the agenda, while keeping in sight the goal of producing a report within three days. Participants were told that while NSF program directors would naturally benefit from such a report, the intended audience would range from new graduate students and postdocs to faculty colleagues, department heads, and university administrators.

The participants separated into two groups: disciplinary and interdisciplinary research. (Tellingly, the panel's reaction to the first subtopic of disciplinary research was a question that would be repeated many times at the workshop: Can, or should, important probability research be separated from at least one application?) Indeed, the two groups were seen to have non-void intersection from the outset. Spirited discussions pertaining to curricular reform, new textbook needs, interdisciplinary training (and protection) of graduate students, start-up overhead, and related topics dominated early sessions, before the division into two groups, which in fact were self-forming. "Protection" might be an unusual word choice; it took on a pointed meaning in the discussions of preparation of future teaching/research faculty for entry into academic departments requiring a disciplinary expertise and desiring interdisciplinary research activity.

High-profile examples related to the massive amounts of microarray data in biology, or to the mathematical evolution of a hundred trillion dollar (and unregulated) derivative security market, make obvious illustrations of compelling mathematical research needs involving probability. Less dramatic but equally important examples point to research frontiers in optimization algorithms, statistical physics, dynamical and physical systems, complex networks, and perception in artificial and natural systems.

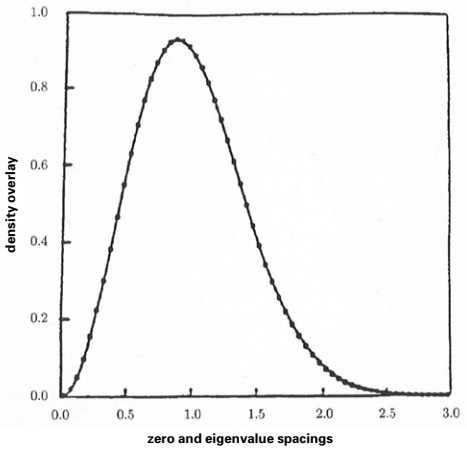

Figure 1. *Nearest-neighbor spacing among 70 million zeros beyond the 10^{20}th zero of the zeta function compared with the Gaussian Unitary Ensemble.*

An important example with far-reaching implications for the physical sciences and the very foundations of mathematics concerns the theory of random matrices, initiated by the physicist Eugene Wigner to study the internal structure of complex atoms. A particularly important topic is the Gaussian Unitary Ensemble of degree N, which consists of the set of all $N \times N$ matrices equipped with the unique probability measure that is invariant under conjugation by unitary matrices and that makes the matrix entries independent random variables. The eigenvalues of a random Hermitian matrix do not behave like independent random variables, and the probability of finding more than one eigenvalue in a short interval is therefore much lower, i.e., the eigenvalues "repel." Evidence has been accumulating since 1970 that the relative spacing of zeros of the zeta function is asymptotically governed by the (explicitly known) spacing of eigenvalues under GUE (see Figure 1). Remarkable advances have been made on related problems, but this problem on the interface of probability and number theory remains open.

You can open *The New York Times* almost any day and read about some complex network, whether as the focal point of a new crisis or as the subject of a success story. Familiar examples include an expanding wireless network that is likely to grow to include not only cell phones but also wireless video and wireless computer connectivity. The Internet and local area networks will continue to expand with their growing importance in business and personal communication. The stability of flow packet designs and protocols will continue to be a highly nontrivial mathematical research problem for decades to come.

Power networks (see Figure 2) have been in the news in recent months, with the frequent blackouts in California and the current scandals involving power allocation and pricing since deregulation. The economic valuation and risk management issues for our natural resources generally involve entirely new market mechanics, which present highly nontrivial challenges to current mathematical theory and computation.

The report provides descriptions of these and other areas of current research. It points to the many roles probability plays within the mathematical sciences, making clear that these are exciting times for probability research, both theoretical and applied.

The report's executive summary includes the following recommendations:

- Universities must find mechanisms to bring coherence to the currently diffuse place of probability on their campuses.
- Mathematics departments should incorporate probability into the core curriculum at all levels.
- Faculty responsible for instruction in probability need to ensure that new curricula reflect recent research developments and the needs of

emerging application areas.

■ Graduate programs must train more researchers in probability, both as a core mathematical discipline and in an interdisciplinary context.

■ Funding agencies and university administrations need to provide the resources necessary to respond to the increased demand for probabilistic methodology.

The report includes several disclaimers. First, the views expressed are not necessarily those of the National Science Foundation. In addition, there was no attempt to be comprehensive, and the examples included reflect the interests and biases of the 18 probabilists who were invited and able to participate. Indeed, we selected participants with a view toward diversity in a variety of categories, such as age, gender, geography, and research interests.

The report was presented in a panel discussion at the annual summer meeting of the Institute of Mathematical Statistics, which was held in Banff, July 31 to August 2, 2002. A significant point of discussion at this session was the notable absence of statistics in the report. Many current research challenges across the sciences involve some combination of modeling randomness, Monte Carlo simulation, and statistical data analysis, all of which depend on probabilistic tools. In particular, statistics and probability are and always have been inextricably linked. A separate report devoted to the vast array of statistical research issues is being prepared in connection with a similar workshop held at NSF in March 2002. For this reason, the panel focused primarly on other roles of probability within the mathematical sciences. The full probability report can be viewed on the Web at http://www.math.cornell.edu/~durrett/probrep/probrep.html or http://oregonstate.edu/dept/math/research/probrep.html.

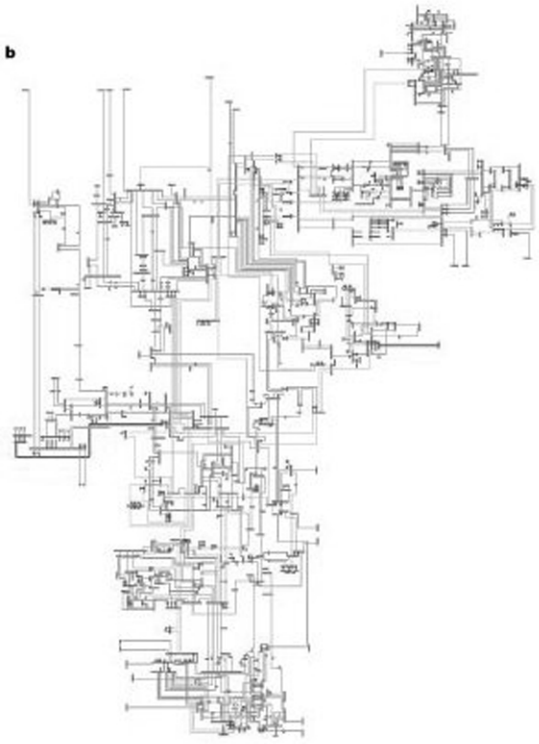

Figure 2. *New York state power grid.*

Probability has clearly arrived, both as a disciplinary core subject and as a central player in a diverse array of modern applications throughout the sciences and engineering. The SIAM 50th Anniversary Meeting in Philadelphia this summer provided further excellent illustrations of the special significance of probabilistic methods in the mathematical sciences today. In addition to a number of special sessions and minisymposia involving probability and stochastic processes, Donald Geman's invited mini-tutorial on computer vision, George Papanicolaou's plenary address on array imaging, time reversal, and communication, Eric Lander's John von Neumann Lecture on the human genome and subsequent research, and the keynote address by NSF director Rita Colwell were all notable for their many references to the key elements of randomness, uncertainty, and chance.

We hope that this workshop report will help to convey some of the pressing needs and wonderful opportunities for research in probability to students, colleagues, and university and government administrators responsible for setting the nation's academic research agenda.

Philip Protter (protter@orie.cornell.edu) is a professor in the Department of Operations Research and Industrial Engineering at Cornell University, and Ed Waymire (waymire@math.orst.edu) is a professor in the Department of Mathematics at Oregon State University.