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Constants in the Universe: Their Validation, Their Compilation, and Their Mystique

The Constants of Nature. *By John D. Barrow, Pantheon, New York, 2002, 352 pages, $26.00.*
Mathematical Constants. *By Steven R. Finch, Cambridge University Press, Cambridge, UK, 2003, 602 pages, $95.00.*

BOOK REVIEW
By Philip J. Davis

Pythagoras is supposed to have said that "numbers are all," and this philosophy of mathematics is known as pythagoreanism. You can call pythagoreanism "numerology," but you might then want to reconcile, if you can, the fact that the roster of numerologists runs from crackpots to the eminent astrophysicist Sir Arthur Eddington.

Judging from the subtitle of John Barrow's book, "The Numbers that Encode the Deepest Secrets of the Universe," *The Constants of Nature* is a pythagorean tract. What's the book like? Barrow, a professor of mathematics at the University of Cambridge, a prolific popularizer and a bit of a mystic, has produced a work that should appeal even to readers with only the fuzziest knowledge of contemporary physics. He has given us a rollicking exposition of the origin and the nature of such physical numbers as the velocity of light, the gravitational constant, the fine structure constant, Eddington's famous 137, the proton–electron mass ratio, Planck's constant, the number of photons per proton, and more. To this, Barrow has added historical, biographical, anecdotal, speculative, and bibliographical material in pleasant and stimulating proportions.

Barrow raises big questions in this book, questions that the sharpest brains have pondered: Can the constants of nature be related and hence reduced in number? Are the constants really constant over time? If the constants were other that what they are, could the universe exist, and if it did, what would its characteristics be? Would that universe support life as we know it? (We can humanize these questions a bit more: If I had had seventeen children instead of four, how would my life have been different?) Why, Barrow wonders, "was Einstein impressed by the ubiquity of small dimensionless numbers in the equations of physics?" How many physical dimensions are there? 4? 17? Can there be a theory of everything, and will such a theory boil down to a few numbers connected by some kind of mathematics, as yet undeveloped? All these questions constitute a mystique of numbers, a scientist's numerology as opposed to the kind associated with speculators on the number 666 in the Book of Revelation.

Having read the author's discussions of these questions, some of which are the frequent subjects of sci-fi and sci-soaps, and putting on my pure mathematician's hat, I would add a few questions of my own: Given that Pythagoras had only a few of the integers and a couple of fractions at his disposal, does it now make sense to ask whether the constants of nature are rational, irrational, or transcendental? Does mathematicians' real number system exist only in their mathematical heads? What is the significance of the steady search for invariance in a world in flux? Which is historically prior, new mathematics or new physics? What is the source of the obsessive quest to reduce the world to as few things as possible? (Occam's Razor at work? Simplicity = Truth = Beauty?) This is philosophical monism, and I tend toward pluralism. I've noticed that the best numerical analysis is often far from simple or beautiful and is not expressible in a few lines of code.

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Steven R. Finch, now a freelance mathematician, has had no difficulty in deciding what the distinguished mathematical constants are. As part (Volume 94) of the series of books published under the aegis of the Encyclopedia of Mathematics and its Applications (Cambridge University Press), Finch has collected hundreds of interesting mathematical constants and written short pieces describing where they come from, who has worked on them, and why they have been considered significant.

Moving rapidly away from the "old chestnuts" such as the gravitational constant, Finch displays a vast forest of constants, arising in a variety of specialties, and which neither I nor most of my readers have ever met up with. Open the book at random (I found it excellent bedtime reading), and you will learn about Favard's constant, Schlüter's constant, Baxter's constant, the Gauss–Kuzmin–Wirsing constant, . . . , and so on into the night.

The book ends with a list of distinguished constants, easily 800 of them, arranged in order of increasing size. One is tempted to contrast this list with *A Dictionary of Real Numbers* by Borwein and Borwein (Wadsworth and Brooks, 1990), which lists 100,000 combinations of values of very familiar functions. Whereas Finch's 800 have come up in substantial contexts, the hope of Borwein and Borwein was that such contexts would materialize for some of the numbers on their list.

In sharp contrast to Barrow's book, *Mathematical Constants* raised no deep philosophical questions in my mind; to mix a metaphor, Finch's forest of constants lies in another ballpark entirely. No questions of validation (although questions of expeditious computation do arise), no cosmological conclusions. No angst. But wait; I must swallow my words a bit. After viewing the wide range of mathematical specialties that gave rise to these constants, I wonder whether there is any field of mathematics that does not generate its own set of distinguished constants.

It was the insight of Archimedes, c. 225 BC, in his Sand Reckoner (the *Arenarius*), that mathematics does not lack for numbers.

Recognizing this, we can view the collecting and elucidating of distinguished constants as an ongoing project—never completed, merely abandoned at some stage. I shall soon submit for Finch's open-ended collection the one constant that I have lifted from obscurity, dubbing it the Constant of Theodorus and shining a bit of light on it; others have studied it in considerable depth.

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The juxtaposition of these two books, bearing the same key word in their titles but really quite different, leads me to compare and contrast how numbers arising in different areas are validated. This is a theme I have harped on before (see, for example, "Count, Recount, and Fuzzy Math," *SIAM News*, January/February 2001, http://www.siam.org/siamnews/01-01/fuzzycount.htm).

Mathematics is traditionally regarded as having arisen from the human desire (or need) to count: one, two, three . . . After counting comes addition: one plus one is two. And from there sprouts the whole of mathematics, both pure and applied. Now the funny thing is this: After the tens table, we are not really able to make direct counts with anything like the fidelity required by rigid logic. Spill a pint of pea beans on the table and count them. You get an answer. If you want to check the answer, you count them again. How many times must you count them, or ask your wife to count them, before you can say that you have the number with incontrovertible certainty?

We read in Barrow that "the number of protons that lie within the encompass of the observable Universe is close to the number 10^{80}." How were the protons counted? The visible universe contains only one galaxy per (10^7 light-years)^3. How are the galaxies numbered? The published estimated age of the universe changes as new observations and theories emerge.

"With 95% confidence, . . . the New York Stock Exchange will probably survive more than 5.2 years, but less than 8,112 years." Here Barrow is quoting Princeton astrophysicist J. Richard Gott, in an article that appeared in *The Wall Street Journal* (January 1, 2000). Gott sei Dank; but what one really wants to know is whether the new president of the Exchange will survive 3.2 years.

Pass from pea beans and galaxies to the population of the United States, which in addition to being a collection of items to be counted is a part of the social world. How can we count the population with the fidelity required by the Constitution and political and social necessities? Not an easy matter! The mathematical statisticians have suggested a number of alternative ways, and they disagree among themselves. The courts then enter the picture and mandate how the U.S. Census Bureau is to proceed. In the last analysis, then, counting becomes a process whose validation and implementation are defined not by mathematicians or social scientists but by lawyers.

Pass from counting people to counting votes, in Florida in the fall of 2002, for example, and more recently in the California gubernatorial recall election. How can votes be counted with fidelity? What is the definition of fidelity in such a case? An accusing finger has been pointed at technology, and lawyers take over again. The matter is challenged in higher and higher courts and settled there. [Readers interested in these questions are encouraged to read Sara Robinson's articles on voting technology in this and the March issue of *SIAM News*.—ed.]

Compare and contrast, then, the manner in which numbers relevant to the physical universe, the mathematical universe, and the social universe are validated and accepted. While no simple description of these processes is possible, we might say that in the physical sciences observation, experimentation, mathematical theory, computation, sense of coherence, and aesthetic and metaphysical grounds all play a role. In mathematics, we have formulas, algorithms, codes, computations, theories of error, checks, and reruns. In the social universe, validation comes from counting, sampling, hearsay, common agreement, computation with models, legislative fiat, and, increasingly, judicial decisions.

Specific numbers are what our scientific, social, economic and political institutions decide they are. As regards the "purest," most cerebral level of all—mathematics—well, its philosophers have always had great trouble in deciding what numbers—in the abstract—really are.

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