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Mathematics as Narrative

Circles Disturbed: The Interplay of Mathematics and Narrative. *By Apostolos Doxiadis and Barry Mazur, eds., Princeton University Press, Princeton, New Jersey, 2012, 570 pages, $49.50.*

First come the symbols, then comes the narrative.—After Bertolt Brecht

Apostolos Doxiadis, mathematician, actor, film maker, is also the author of the novel *Uncle Petros and Goldbach's Conjecture*, so it is no surprise that he is interested in mathematics and narrative. Likewise for Barry Mazur, prolific author of a large variety of popular mathematical narratives, including the 2004 *Imagining Numbers (particularly the square root of minus fifteen).*

BOOK REVIEW

By Philip J. Davis

Circles Disturbed, a collection of 15 articles by as many authors, is so rich in content that I have found it impossible to do it justice in my allotted space. I give *Circles* a five-star rating and encourage my readers to immerse themselves in it. Though it may not be cricket, I conclude this review by pretending to be a sixteenth contributor to the book, presenting a few of my own observations on the interplay between mathematics and narrative.

Narrative-free presentations of mathematical material are easy to find. When I took second-year calculus as an undergraduate, we were advised to buy a copy of B.O. Peirce's *A Short Table of Integrals*. My edition lists 938 formulas, with hardly any narrative to expound them, to say why they're interesting or why a serious student of mathematics should pay any attention to them. I recall that Dr. John Smyth-Smith (pseud.), a brand new hot-shot TA who later became a prominent algebraic topologist, pooh-poohed Peirce's *Table*, not because it lacked narrative, but because it represented a kind of mathematics that he felt was beneath contempt.

Value judgments about mathematics are aired and enshrined in oral, written, unwritten, often unconscious, narratives that waft through our increasingly mathematized culture. They range from simple approval/disapproval to long and detailed depictions of, say, *17 Equations That Changed the World*.*

J.L. Walsh, one of my professors, said to me years later when we were writing a joint paper, "Ideally, every mathematical expression ought to be embedded in an English-language sentence." This was Walsh's beau ideal of mathematical expository style. I've kept his admonition in mind over the years, hoping to adhere to it. But looking over some of the mathematical material I've produced, I find that I pretty much abandoned it.

Take a look at the chapter on the gamma function that I wrote for the famous Abramowitz and Stegun *Handbook of Mathematical Functions*. It introduces the gamma function by Euler's integral, and it's full of isolated identities, such as $\Gamma(1 + iy) = iy\,\Gamma(iy)$. But there are no accompanying narratives.

Timothy Gowers, one of the *Circles* authors, illustrates that while the same mathematical fact or theory can be introduced in a variety of ways, some presentations will be distinguished for their vividness. It's not easy to recover what was in my mind more than a half century ago, but having produced my chapter in Abramowitz and Stegun, I must have felt a crying need for an embedding narrative that would produce vividness, for I went on to write *A Historical Profile of the Gamma Function* (*The American Mathematical Monthly*, December 1959).

Thus, I agree heartily with David Corfield's suggestion, in the chapter "Narrative and the Rationality of Mathematical Practice," that though mathematics is traditionally considered the logical discipline par excellence, to be fully rational, mathematicians must embrace narrative as a basic tool for understanding the nature of their discipline and research.

Mazur presents and discusses a possible taxonomy of narratives: (1) origin stories, (2) purpose stories, (3) "raisins in the pudding," (i.e., the purely ornamental), and (4) dreams, their consequences and motivations. Interested mainly in (4), Mazur considers what it means for a mathematician to have a dream: "Kronecker's dream is to provide us with some way of explicitly understanding *fields* of *algebraic numbers* that are *abelian* over a given *number field*." I suppose that narratives of all these types can be found in current expository mathematical material.

Consider mathematical narratives that attempt to interpret very old texts. Methodologies of interpretation go by the fancy name "hermeneutics," and a frequently utilized interpretive device goes by the name "present-centered history." The interpreter in the latter describes the past using full knowledge of developments of subsequent importance. But can the interpreter do better than that and really get into the minds of the past creators? Only imperfectly. Eleanor Robson, known for studies of ancient Babylonian mathematics, has suggested as much. Nonetheless, efforts have been made in this direction; Robson referred† to the work of Henk Bos and Herbert Mehrtens, who considered the relation between mathematics and the enveloping society, and of David Bloor, for whom mathematics was a social construction tout court.

The creation of new mathematics is often said (perhaps fallaciously) to be limited to the young. The later careers of research mathematicians read like a salad bar of biographical possibilities. In my own case, having produced N technical papers in the manner accepted by the research establishment, but not wanting to abandon the subject that has yielded me much bread, butter, and pleasure, I switched over to mathematical history, philosophy, metaphysics, pragmatics, and even journalism and interviews. In these areas narrative reigns supreme. What follows is an instance of this sort of narrative.

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On the basis of the following characteristics, I see the Unity of Mathematics as a dream, a chimera, an ideal.

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*Ian Stewart, *In Pursuit of the Unknown: 17 Equations That Changed the World*, Basic Books, 2012.

†*Mathematics in Ancient Iraq: A Social History*, Princeton University Press, 2008.

■ The subject classification scheme. To some extent, each of the close to a hundred mathematical areas has its own techniques, intellectual resources, and devotees, although there may be tenuous connections between, say, potential theory and non-associative algebras, and rare collaborations between experts that indicate a certain degree of unity and coherence in the field of mathematics. I find the lack of unity more strikingly located elsewhere.

■ Diachronic and cross-cultural disunity. Written mathematics is easily 4000 years old. It has been created by people and has served a variety of purposes for them. A mathematician lives in a sub-culture at a certain time and place. A piece of mathematics does not exist only as a sequence of special symbols, because the naked symbols are essentially uninterpretable. The symbols are embedded in a cloud of knowledge, meanings, associations, experiences, imaginations that derive from the particularities of time, place, person, and the enveloping society.

Pythagoras asserted that 3 is the first male number. In certain Christian theologies, 3 is the number of the Godhead. If, in Indian numerology, the numbers 1,10,19, and 28 are "ruled by the sun," the meaning of and the belief in those words may, as Eleanor Robson would suggest, escape my readers.

■ Semantic ambiguity. I could write down the sequence of symbols x ╚$\cap\sigma\Sigma \equiv 6$ and claim that it is a piece of mathematics. But this claim could not be substantiated solely on the basis of the symbols. To provide meaning, every mathematical statement must be embedded in a narrative in some natural language (English, German, et alii). Furthermore, its significance as mathematics cannot be established if knowledge of it is limited to one person as a private revelation.

■ Semiotic ambiguity. Can it be determined when two mathematical statements, phrased differently, assert the same thing? Barry Mazur begins a discussion of this question in the book under review in a chapter titled "Visions, Dreams, and Mathematics."

■ Non-acceptance or doubts about certain theories put forth by professional mathematicians. Examples are easily found. Originally, there was one formal geometry: that of Euclid. After Bolyai and Lobatchevski, there were three, and since Riemann, we have had an infinity of geometries.

Zermelo did not believe Gödel's proof. For George Berkeley, infinitesimals were the "ghosts of departed quantities." Skepticism regarding the concepts of Cantor has been expressed by many, including Kronecker, Poincaré, Zermelo, E. Picard, Brouwer, Hermann Weyl, Wittgenstein, Errett Bishop.

A well-known quote from the great applied mathematician Richard Hamming sums this up:

"I know that the great Hilbert said 'We shall not be driven out of the paradise that Cantor has created for us,' and 'I reply I see no reason for walking in.' "

■ Philosophic ambiguity. Prior to the end of the 19th century, there was one philosophy of mathematics: platonism. Now there are easily five distinguishable philosophies, together with variations that exhibit the Freudian "narcissism of slight differences."

And yet. . . . There is a unique corpus of material, a creation of human intellect that is called mathematics, in which narrative plays a vital role. But tell me, in what does the often touted unity of mathematics consist?

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The universality of mathematical narrative. Atsuko Tanaka (1932–2005), in her 16mm film Round on Sand *(Japan, 1968). Copyright 2010 Ryoji Ito and Takehiro Nabekura, courtesy of the Ashiya City Museum of Art & History. Photo by Takehiro Nabekura.*