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Imaging Science

Imaging with Ambient Noise

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In sensor array imaging, waves are used to probe an unknown medium. Different wave transmission and reception modalities can be implemented, depending on the application: nondestructive testing, medical imaging, seismology. Travel time tomography, backpropagation, and migration are well-known techniques for extracting information from array data. Recently, new ideas have emerged from experimental observations, such as super-resolution effects in time-reversal experiments, and from theoretical concepts, such as coherent interferometry or cross-correlation methods. In particular, the possibility that uncontrolled ambient noise sources can be substituted for controlled active sources has attracted a lot of attention, especially in seismology.

The geological structure of the Earth can be estimated by observing the propagation of elastic waves in the Earth's crust. In classical seismology only waves generated by earthquakes or by artificial explosions and recorded by a network of sensors (geophones) are used. Even in the absence of earthquakes or explosions, however, the sensors record a weak incoherent signal: the seismic noise, whose frequency components, around 0.1 Hz, consist mostly of surface waves produced by the interaction of the ocean swell with the coast. It turns out that these noise signals can be used to obtain travel time information.

The main observation is that the time cross correlation of the noise signals recorded at two sensors is related to the Green's function of the wave equation between these two points. As a consequence, the travel times between pairs of sensors can be estimated by computing the cross correlations of the noise signals recorded by the sensors. The background speed of propagation can then be estimated from the travel times between sensors in a network covering the region of interest, as done in southern California [4].

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In fact, the idea of exploiting ambient noise and using the cross correlation of noise signals to retrieve information about travel times was first proposed in helioseismology [3]. The idea has since been applied to background velocity estimation from regional to local scales, as well as to volcano monitoring and petroleum prospecting [2].

These observations have led to novel ideas for imaging. First of all, drawing on simple arguments, it is quite easy to explain the relation between cross correlations of noise signals and the Green's function. Indeed, when the support of the random noise sources completely surrounds the region of interest, it follows from the Helmholtz–Kirchhoff identity that the derivative of the cross correlation of the recorded signals is the symmetrized Green's function between the sensors.

In many applications, however, the noise source distribution is spatially limited and the waves recorded by the sensors are dominated by the flux coming from the direction of the noise sources. The cross correlations of the recorded signals then depend on the orientation of the sensors relative to the direction of the energy flux. This significantly affects the quality of the estimate for the Green's function. As can be explained by stationary phase arguments, it is good when the line between the sensors is along the direction of the energy flux and bad when the line is perpendicular to the flux.

When there is multiple scattering in the medium, as in an ergodic cavity or in a randomly inhomogeneous medium, the directional diversity of the recorded signals is enhanced by multi-pathing. This was studied in the context of time-reversal experiments, and it has been shown that time-reversal refocusing can be enhanced in a randomly scattering medium. It turns out that cross correlations of ambient noise signals can be used to exploit the enhanced directional diversity of the multiply scattered waves for travel time tomography. Indeed, the travel time between two sensors can be estimated even in unfavorable situations, provided that special fourth-order correlation functions with auxiliary sensors are used [5]. An unfavorable situation is one in which the main component of the energy flux from the noise sources is roughly perpendicular to the ray connecting the two sensors. The fourth-order correlations can enhance travel time estimation between the two sensors by exploiting the scatterers as secondary sources and by reducing the contribution of the primary flux.

Ambient noise signals can also be used for passive sensor imaging of reflectors embedded in the medium. Indeed, in the presence of reflectors, the cross correlations between any two sensors have, in addition to the main peaks of the travel times between them, other peaks at lag times related to travel times from the sensors to the reflectors. Analysis of the relation between the secondary peaks in the cross correlations and travel times between sensors and reflectors shows how, through suitable migration of the correlations, the reflectors can be imaged. The resolution of the image depends on the directional diversity of the noise signals relative to the sensor array and on the reflector location. When directional diversity is limited, it is possible to enhance it by exploiting the scattering properties of the medium. Scattering increases the fluctuation level of the cross correlations, however, and therefore blurs the image. Evaluating the trade-off between resolution enhancement and reduced signal-to-noise ratios as a result of scattering—a central question in this domain—requires a deep understanding of wave propagation in random media and scaling issues. Following the coherent interferometric imaging ideas developed for broadband deterministic pulses [1], it is possible to form space–time local cross correlations of the cross correla-

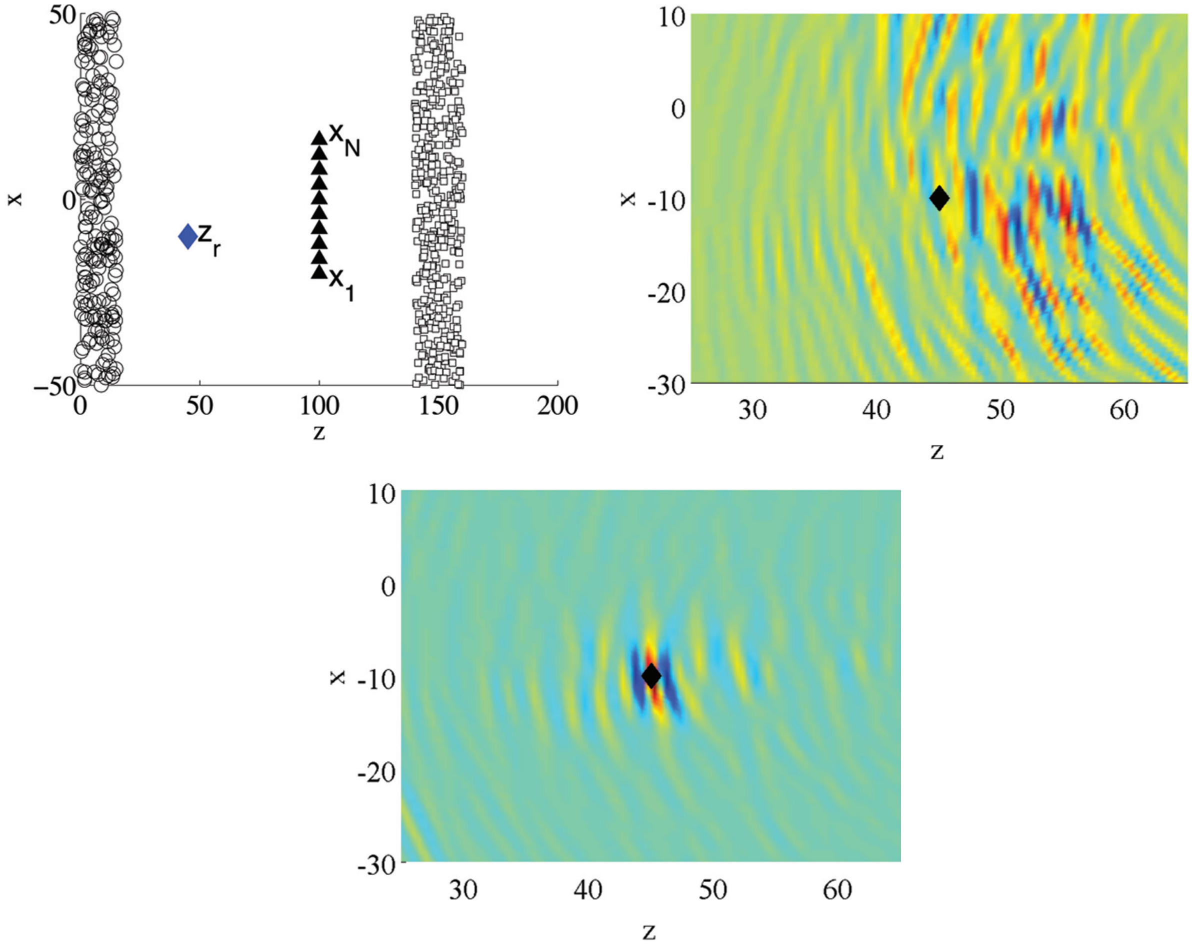

Figure 1. *Reflector imaging from signals generated by ambient noise sources. Top left, plot of the configuration, showing noise sources (circles), scatterers (squares), sensors (triangles), and a small reflector (diamond). Top right, image obtained by migrating the matrix of cross correlations of the recorded noise signals. Bottom, image obtained by migrating special fourth-order cross correlations.*

tions and migrate them. The use of iterated cross correlation can dramatically enhance the quality of the images (see Figure 1). These techniques are new and could certainly be improved and extended to other areas, such as microwave imaging, and they could also trigger new ideas in active array imaging.

References

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For Further Reading

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