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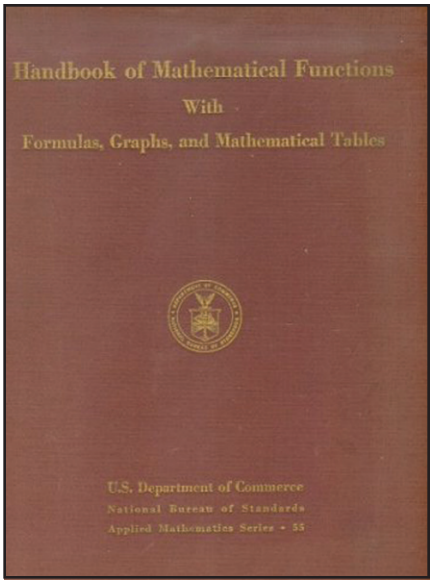
Formulas Galore

NIST Handbook of Mathematical Functions. *Edited by Frank W.J. Olver (editor-in-chief), D.W. Lozier, R.F. Boisvert, and C.W. Clark, National Institute of Standards and Technology, Gaithersburg, Maryland, and Cambridge University Press, New York, 951 + xv pages and a CD, 2010, $99.00 hard cover, $50.00 soft cover. The online version, the NIST Digital Library of Mathematical Functions, DLMF, can be found at http://dlmf.nist.gov.*

BOOK REVIEW

By Philip J. Davis



"Abramowitz and Stegun," created at the National Bureau of Standards and released in 1964.

Abramowitz and Stegun

The compilation of mathematical material has been going on as long as mathematics itself has been on the intellectual horizon. Among the many modern compilations, one that has ranked high in terms of authority, completeness, and utility is the *Handbook of Mathematical Functions*, aka "Abramowitz and Stegun," aka "AMS 55." The immediate origin of this effort was the Mathematical Tables Project, a Rooseveltian WPA project that began operation in 1938 in New York City. As early as 1952, Milton Abramowitz—inspired, I suspect, by the 1909 (and subsequent) editions of *Funktionentafeln mit Formeln und Kurven* of Eugen Jahnke and Fritz Emden, and by the multivolume *Higher Transcendental Functions* (the Harry Bateman Project)—proposed a work that ultimately became the *Handbook of Mathematical Functions*.

Abramowitz lamentably passed on before the project had been completed, but the work was successfully brought to fruition by the untiring efforts of Irene Stegun. The Handbook was created at the National Bureau of Standards and produced and issued in 1964 by the U.S. Government Printing Office at a price of $6.50.

"Abramowitz and Stegun," to which easily 30 international authorities contributed, was a bang-up success. In August 1969, the 100,000th copy of the book was printed; as the author of two chapters, I received the 100,016th copy as a gift from the director of NBS. I have only a rough idea of how many copies were ultimately sold. The government edition, along with other editions that we authors considered shamefully pirated (but that, legally speaking, were not), probably added up to several hundred thousand copies.

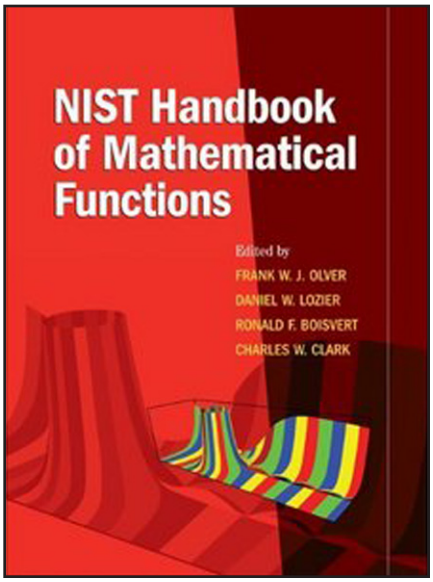
I recall discussing plans for the book with Abramowitz and telling him that I would contribute a chapter on the gamma function. My interest in the gamma function goes back to a course in advanced calculus I took in my junior year from David V. Widder. As a reading-period assignment I wrote an essay on the gamma function regarded as the complex analytic interpolatory extension of the sequence of factorials 1, 1×2, $1 \times 2 \times 3$, ...

In the course of composing my chapter for the Handbook, I became interested in the history of the gamma function. I recall chasing after a reference* that took me into the vast stacks of the Library of Congress. A further offshoot of this interest and search was the article "Leonhard Euler's Integral: A Historical Profile of the Gamma Function," which appeared in the *American Mathematical Monthly* in December 1959 and for which I won the Chauvenet Prize of the Mathematical Association of America in 1963. Though the SEAC (Standards Electronic Automatic Computer) had been around since 1950, I recall checking some published tables on a mechanical desk computer.

In 1988, NBS morphed into NIST. The NBS Handbook has now morphed into the NIST Handbook and I have morphed into my anecdotage. Enough by way of personal reminiscences!

The NIST Handbook

More than a half century has passed since the various authors worked on the NBS Handbook. A good many of its pages were devoted to numerical tables, but there is now hardly any need for extensive tabular values. The half century has been a time of revolutionary changes in the manner in which mathematics is pursued and communicated. During this time, scientific software and computer graphics have burgeoned, new special functions and applications have appeared, and dissemination outlets have multiplied. For these reasons and for numerous others, it became crystal clear about a decade ago that it was time to update the NBS Handbook. As the entire book is available online (DLMF, the NIST Digital Library of Mathematical Functions, which has a different format and many advanced Web capabilities), there is no need for me to describe its contents. What follows consists largely of a few impressions.



The manner in which an update would be undertaken turned out to be a nontrivial exercise in planning and decision making—whom to ask for financial support, what authorities to contact for individual chapters, when to say "enough is enough" when functions, formulas, or identities have multiplied like rabbits. For example: Five zeta functions are described in the NIST Handbook, but the Epstein zeta function is not among them. Should it have been included? In view of the existence of search engines, what was to be made available online and what in hard copy? And as regards the final product, what formatting ideas would be solicited and put in place? Would the material be in the public domain, or

*P.H. Fuss, ed., *Correspondance Mathématique et Physique de Quelques Célèbres Géomètres du XVIII*ème *Siècle, Tome I,* St. Petersbourg, 1843. (Incidentally, rare book dealers offer this book for several thousand dollars.)

would it be protected by copyright? The options suggested by these questions had to be winnowed. In the end, NIST holds the copyright to the printed book, and the online version is freely available to the public.

The NIST Handbook is a handsome product, with large pages and large type. The book is quite heavy; for convenience, one might be inclined to place it on a stand, as with an unabridged dictionary. The book contains numerous graphics, almost all in color. References and cross references to books and articles abound. Applications to both the mathematical and physical sciences are indicated.

To show the degree of expansion from the NBS Handbook, I cite two chapters. The original chapter on the gamma and related functions contained eight sections plus tables; the updated chapter, by R.A. Askey and R. Roy, has 24 sections. The updated chapter on number theory, by T.M. Apostol, has 22 sections, as compared with 10 in the original.

A quote from chapter 17, "q-Hypergeometric and Related Functions" (section 17.18), illustrates the "flavor" of the book's recommendations for computation:

"The two main methods for computing basic hypergeometric functions are: (1) numerical summation of the defining series given in §§17.4(i) and 17.4(ii); (2) modular transformations. Method (1) is applicable within the circles of convergence of the defining series, although it is often cumbersome owing to slowness of convergence and/or severe cancellation. Method (2) is very powerful when applicable (Andrews (1976, Chapter 5)); however, it is applicable only rarely. Lehner (1941) uses Method (2) in connection with the Rogers–Ramanujan identities."

The NIST Handbook is indeed a monumental achievement, and the many, many individuals who participated in its creation and dissemination are to be congratulated and thanked. One should take special note of the editorship of Frank W.J. Olver, emeritus professor of mathematics at the University of Maryland, for whom this book is the fruit of his lifelong study and concern with special function theory, and of Daniel W. Lozier, who leads the Mathematical Software Group of the Mathematical and Computational Sciences Division at NIST.

Still, as it might have been written in Ecclesiastes, "Take heed, my child, for the writing of handbooks there is no end."

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