Selecting Our Leaders

Mathematics and Democracy: Designing Better Voting and Fair-Division Procedures. *By Steven J. Brams, Princeton University Press, Princeton, New Jersey, 2008, 373 pages, $65.00.*

"Is Dis a System?"

—Milt Gross, *Nize Baby* (1926)

Grown step by step in response to changes in population, education, income, moral and ethical criteria, technology, and the media, the U.S. presidential election system might be viewed as the best of all possible systems, in both the Leibnitzian and the Darwinian senses. If so, why do we grouse when we've got what we deserve?

BOOK REVIEW
By Philip J. Davis

I've thought of a better system. It is said that, existentially, somewhere in Missouri is a little old lady, hidden from view and now approaching 100, who has voted for the winner in every presidential race since Roosevelt defeated Alf Landon. All that's needed is to locate her and ask her to identify the next president. (This is a much sharper version of the principle known as "As Maine goes, so goes the nation.") Having expounded in my teaching career the mysterious ξ that occurs in the Mean Value Theorem, I experience no angst in the presence of existential questions. My system would save billions in providing us with a president, but the media, the lobbies, the manufacturers of voting equipment would not go along with it. Neither would you, and I suspect that you'd place me in the same bin as those who would replace the predictions of the National Oceanic and Atmospheric Administration with those of Punxsutawney Phil.

I picked up Steven Brams's book hoping that he, a lifelong expert on voting systems, would propose something better. I even exchanged a few words with him by e-mail. But I was disappointed. More about this later.

Voting theorists have now been plying their craft for three centuries and have devised many systems. Prominent early names include the marquis de Condorcet (1743–94) and Jean-Charles de Borda (1733–99), who proposed methods and discussed their pros and cons.

Brams, a professor of politics at New York University, is a game and voting theorist who has given us many volumes on elections and allied topics. His book is not about the vast paraphernalia and ritual life in which our elections are now embedded, but simply about how we are allowed to mark our ballots. Bram begins with the question, How shall we select a single winner when there are multiple (especially three or more) candidates?

There are now at the very least ten systems for selecting a single winner from a slate of candidates. What, over the years, has created such a plethora? It turns out that numerous systemic axioms, considered as quite natural desiderata when taken in their entirety, are logically inconsistent. This gives rise to a veritable salad bar of paradoxes and dilemmas. (I suspect that paradoxes may outnumber theorems.) A cadre of supporters for each system develops as a consequence, and the heat that accompanies advocacy can be considerable. Surprising? No. Whoever said that life is logically consistent?

These systems can be evaluated as to how they make out with respect to a handful of desiderata. In his first couple of chapters, Brams concentrates on two systems: plurality voting (PV) and approval voting (AV). PV is surely the most frequently employed. On election night we follow PV on TV with bated breath as the late returns from the outlying districts come in and the numbers change.

One trouble with PV is that in a race with three or more candidates, the winner might be the least acceptable to the majority of voters. Minority candidates can easily become "spoilers." Is this a paradox or simply a drawback?

In AV a voter can vote for (i.e., approve) as many candidates as he/she wishes. The winner is the candidate with the most votes. AV is Brams's beau idéal, and he piles up six reasons for its superiority. All that is needed is the implementation, the move from the idea to the polls, and this, as T.S. Eliot noted, is where the shadow falls. One might think that scientific societies would be attracted to Brams's argumentation; the AMS and the MAA have indeed adopted AV, but such is not always the case with societies.

SIAM, for instance, has a mixed system. Jim Crowley, executive director of SIAM, wrote to me:

"Our Board has had discussions on this—should we allow preference voting, for example?—but in the end the Board retained the voting method contained in the original bylaws. For each contested office, there are two candidates and a voter can vote for one. In the case of Council elections, where there are four open seats and eight candidates, a voter can vote for at most four."

Brams goes on to discuss combined approval and preference (PAV) systems and the "fallback" voting (FV) system. He describes the latter, which is less familiar, as follows:

"Voters rank only candidates of whom they approve. The winner is determined according to the majoritarian compromise (the MC), except that the descent process stops for a voter when he or she ranks no candidates lower."

Here we need another definition: MC, Brams writes, is a

"voting system in which first-choice votes, then second-choice votes, then lower-choice votes are counted until at least one candidate receives a majority of votes; if more than one candidate receives a majority, the candidate with the most votes wins."

My head is whirling. But not to worry: Computerized election equipment, even as its existence allows very complex voting systems to emerge,

will give us the names of the winners immediately. For elections with multiple winners, other systems compete—for example, CAV (constrained approval voting) or minimax procedures. Some questions are discussed, quite naturally, from a game-theoretic point of view: How do or should coalitions form? How should cabinet ministries be formed?

Finally, in five successive chapters, Brams turns to the pressing question of how a cake can be cut up equitably. (Or, as King Solomon might have said, how the baby can be divided. Would you believe that in the past quarter-century at least seven technical authors have treated Solomon's decision (I Kings, 3:16–28) to split the baby in half?)

Mathematics and Democracy is rich in analyses of historical cases. Among other cases, you can read an analysis of the voting in the U.S. House of Representatives on the Wilmot Proviso (1846), which prohibited slavery on land acquired in the Mexican War. There are analyses of coalition formation among the justices in a more recent (1973) Supreme Court case regarding obscenity and in a 1974 case regarding the notorious White House tapes. The Yom Kippur War (1973) and the Camp David Accords (September 1978) are scrutinized. Should our politicians and diplomats be trained in game theory?

For the casual reader, a glossary of terms and chapter-by-chapter summaries of the author's conclusions are very useful. More technically oriented readers can chew on such concepts as Nash equilibria, Pareto voting, (Richard) Hamming distance, maxi-min allocations, and techniques for rounding off fractions in apportionment problems. Those who wish to follow Brams's arguments even more closely will be thrust into combinations, permutations, probabilities, and sequences of inequalities that represent states of desirability, or the reverse.

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I now return to my disgruntlement with the entire presidential election scene in the U.S. I have felt, and am sure some of my readers have felt, that the process of electing a president, having become so lengthy, so costly, so bizarre, illogical, Byzantine, having maintained itself despite its own weight and follies, ought to be pulled down. Yes, pulled down caucus by caucus, primary by primary, convention by convention, and replaced by something simpler and saner. Alas, this is an unlikely event. I sought Brams's views. What did he favor? He answered me as follows:

"You ask a good question. It's easier to say what I do not favor—single national party primaries—because they would all but rule out candidates who are not already nationally prominent or very rich. Regional primaries, perhaps spaced a month apart with their order determined randomly, seem to me a more reasonable solution.

"But I actually think there is a logic to the present system, because the campaign stress test that it forces candidates to undergo tends to weed out weaker candidates. Of course, it's purely tradition that Iowa and New Hampshire go first, so the choice of early states—in principle, but probably not in practice—could be varied. This weeding out, however, does not always work: A Goldwater (in 1964) or a McGovern (in 1972) can win his party's nomination and then, because he is an extremist, be a disaster to his party in the general election.

"I believe approval voting . . . would tend to prevent such nominees, and favor more centrist candidates, if used in party primaries."

Read *Mathematics and Democracy*: You will learn of the vast number of voting options that have been mooted, and you will easily conclude that any proposed change, however minor, will arouse fury in some constituency somewhere. Nonetheless, the systems that are in place do change and develop as time and circumstances compel and allow. Continuing along current lines, the presidential election seems to be developing into the system that will attract the highest media ratings. For—let's face it—political voting in the USA is no longer just a social obligation: It has become a vast industry.

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