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Updating "Abramowitz and Stegun"

By Philip J. Davis

The organized accumulation and dissemination of mathematical information probably date back to the origins of the subject itself. "The first securely debatable mathematical table in world history comes from the Sumerian city of Shuruppag circa 2600 BCE."* Fast forward (very fast) four and a half millennia, skipping thousands of mathematical tables, and we come to the famous *Handbook of Mathematical Functions* (known popularly as Abramowitz and Stegun) issued by the National Bureau of Standards in 1964. A&S was an instant best seller, and with a million copies now in print, it has been one of the most frequently referenced works in the whole scientific/technological literature.

As an employee of NBS in the 1950s, I joined the crew that produced its 29 chapters; I also contributed two chapters, one on the gamma function and the other on numerical interpolation, differentiation, and integration. Those were the days of the first-generation electronic digital computers (SEAC, May 1950, et alii), but I recall that to supply certain mathematical tables, we were still using desktop electric machines, such as Marchants and Friedens. Even as we contributors to A&S collected, compiled, edited, computed, checked, and prepared our work for publication, we were aware that information technology was moving so rapidly that our work would be born with certain features already perceived as greatly improvable or obsolescent.

On scanning the original A&S, what one notices above all is the vast proportion (more than 50%) of its 1046 pages devoted to tables of all kinds, e.g., the error function er*f*(*x*) or the elliptic integral of the third kind. These inclusions can be seen in the context of the times and their computational practices. A great number of special functions were computed in pre-electronic days, beginning in antiquity and continuing to the mid-1940s. At that point, the facility of the early generations of electronic digital computers was responsible for a euphoric spate of tabulated values, such as Bessel functions. Such tabulations, having been replaced by algorithmic software, are now dead as doornails—well, not quite dead.

In the half century since A&S appeared, absolutely remarkable facilities and potentialities have become available, all of them relevant to a presumptive update of A&S. To mention just a few: personal computers of enormous power, editorial and formatting software, the Web, search engines, specialized language packages, and software like MATLAB and its associated toolboxes; all continue to increase in scope, versatility, and utility.

Under the direction of a board of editors consisting of four principal editors and several associate editors, NIST (the National Institute of Standards and Technology, as NBS was renamed in 1988) has just released a general outline of an update of A&S—a work known as the *Digital Library of Mathematical Functions* (DLMF)—together with five representative chapters. Daniel W. Lozier is the general editor; the other three principal editors are Frank W.J. Olver (mathematics), Ronald F. Boisvert (information technology), and Charles W. Clark (physical applications). The topics of the representative chapters in the current release, accessible at http://dlmf.nist.gov, are asymptotic approximations, the Airy and related functions, the gamma and related functions, number theoretical functions, and the Wigner 3*j*,6*j*,9*j* symbols. The editors hope to receive feedback from the community on the released material.

In the last half century, there have been theoretical developments in the area of special mathematical functions, as well as many new applications, all crying out to be taken into account. In view of this totally new and upgraded environment, the planners and contributors to an update of A&S faced numerous problems.

With seed money from the National Science Foundation, together with substantial internal funding from NIST, the work is the product of more than fifty international subject-area authorities. Each of the 36 chapters has been through many carefully edited preliminary drafts and independent validation processes.

Scanning the five chapters supplied, I noticed that in addition to the mathematical formulas, they include advice on methods for computing the functions given, as well as pointers to available software. The text also differs from A&S in that its frequent links provide references to or hints for the proofs of all mathematical statements. In the five chapters released, there are numerous versatile interactive visual aids, including three-dimensional representations that will yield qualitative information on the behavior of mathematical functions. In addition to linked bibliographies and indices, there is a built-in mathematical search engine applicable to the whole of the text. It is clear that constructing such an engine necessitated the development of special coding features over and above what can be found in Google and its peers. Anticipated further work on the engine will extend its applicability beyond DLMF.

I have been informed by the project editors that the Web version will be complemented by a hard-copy version of about a thousand pages, which represents twice as many pages of technical information as in A&S. Each version (Web and hard-copy) has obvious advantages and limitations.

The initial printing of A&S was done by the Government Printing Office, and the work was in the public domain. NIST, under an applicable provision of the U.S. Code, will hold the copyright to DLMF.

In the last half century, authors, working alone or in small groups, have produced hard-copy mathematical handbooks covering a variety of areas. During the same time, there have been theoretical developments in the area of special mathematical functions, as well as many new appli-

* Eleanor Robson, in *The History of Mathematical Tables: From Sumer to Spreadsheets*, M. Campbell-Kelly et al., eds., Oxford University Press, 2003.

cations, all crying out to be taken into account. In view of this totally new and upgraded environment, the planners and contributors to an update of A&S faced numerous problems: how best to take advantage of the new facilities, how to expand the coverage, how—given that the amount of mathematical information “out there” is quasi-infinite—decently to limit the coverage, and which of the possible computer bells and whistles to include and which to suppress. The presentation of material and its formatting would have to be comprehensible to non-specialists. Together, these considerations most certainly required compromises to the personal agendas and pet preferences of the contributing authors. Thus, an enterprise of this scope reflects the knowledge and experience and scientific interests of the working group and carries with it a special flavor and utility. I admire the courage and success of the planners and their contributors in consolidating their ideas; they have arrived at a product that should be tremendously useful.

The story that began in Shuruppag forty-five hundred years ago is an open-ended one. Once the completed version of DLMF (release expected in 2009) is in hand, one will be able to dream and speculate on updates to it. Will new and important special functions emerge? New informational facilities? Will the work be constantly updated, as is now the case with a variety of packages? Insofar as the computer is not merely a useful aid to mathematics and science, it represents a meta-development that might very well change the nature of what mathematics is considered to be, how it is developed, and how it is used. Consider, for example, the familiar identity

$$e^{i\theta} = \cos(\theta) + i \sin(\theta).$$

Its components easily span two and a half millennia of mathematical development, interpretation, and application. Into what theoretical and technological environment, into what milieux of thought and action, will such pearls of special function theory be found in the year 2050? This question, though difficult, is not beyond all conjecture.

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