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# Dense Granular Systems: From Theory to Applications

*By Lou Kondic, Corey S. O'Hern, and Robert P. Behringer*

Granular materials appear in a host of applications, ranging from the handling and transport of rocks and soils in the oil and mining industries to the processing of pharmaceutical fine powders. They also play important roles in geophysical phenomena, e.g., mudslides and debris flows, and in astrophysical phenomena, e.g., planetary formation. Despite their wide-ranging appearance in industry and nature, our ability to predict the behavior of granular materials lags far behind that for more conventional materials like Newtonian fluids. Industrial devices used with these materials operate well below designers' expectations, and failures, which are often catastrophic, costly, or both (see Figure 1), occur far more frequently than for fluid-handling devices.

**Figure 1.** *The collapse of a silo is an example of the consequences of large stress fluctuations.*

Researchers have devoted intensive efforts to the behavior of granular materials, which can exist in dilute gas-like phases, solid-like phases, and transitional fluid-like phases. The session at ICIAM '07 titled "Dense Granular Systems: From Theory to Applications" focused on the dense transitional phase, in which grains maintain substantial long-term contacts and can deform, but in which the system still flows. This regime, which exhibits strong fluctuations but clearly cannot be described as a granular gas, is one of the most challenging areas of current research.

In the case of granular gases, it is fluctuations in the grain velocities that determine the key physical quantities in a system. In fact, kinetic theories with the thermodynamic temperature replaced by the granular temperature $T_k \propto \langle v^2 \rangle$ can produce accurate calculations of the density and pressure profiles in driven granular gases.

In the dense, slowly flowing regime, connections between fluctuations and larger-scale behavior have yet to be determined, although studies have shown that force and velocity fluctuations can be very important at scales considerably greater than the size of a grain. In the dense state, forces are carried on filamentary structures known as "force chains," which can have relatively long-range spatial correlations. Figure 2 (see page 2) illustrates force chains obtained via discrete element simulations in a sheared granular system subjected to time- and space-dependent perturbations at one of the system's boundaries. When dense granular systems deform, changes in these structures can lead to large force fluctuations, which are often comparable in size to the mean. These large force fluctuations, in turn, influence the stress tensor. Unlike dilute granular gases, in the dense regime there is no established way to characterize these fluctuations and then predict their impact on the macroscopic behavior of the system.

Perhaps one reason for the lack of a generally valid theoretical approach is that granular systems are not an exact fit for any of the three common states of matter. The limits of applicability of commonly used theoretical approaches are also often unclear. Kinetic theory, for example, has been used successfully to explain numerous features, such as clustering of rarefied granular systems, but it cannot be used to describe dense granular flows. Because the granular particles in these systems are in (almost) continuous contact, one could argue that continuum theories, such as elasticity or isostaticity theory, should be more appropriate. Significant difficulties and challenges emerge in applying these theories, however. One such issue (and perhaps the most fundamental) is that continuum theories are based on the concept of averaging over a volume that is large compared with the particle scale, but small compared with the size of the system. *In granular systems, such a volume may not exist.* For this reason, a significant amount of research in the field of dense granular systems is carried out via discrete element simulations, which model individual particles and their interactions.

New mathematical research is required for progress in the area of dense granular flows, in particular for determining the connection between fluctuations and macroscopic behavior in these systems. We need to be able to bridge the scales and formulate continuum models that would provide at least a basic understanding of macroscopic behavior, while including the particle-scale physics. This need is particularly obvious when we consider one of the most fundamental questions about dense granular systems: How does information propagate from one part of a system to another?

The many models that have been proposed differ even in their most basic mathematical properties. Some models, based on a probabilistic picture for force propagation, predict diffusive (parabolic) propagation of forces. Other probabilistic approaches predict a response that can be either wave-like or elastic, depending on model parameters. Some experiments, however, seem to be consistent with an elastic response to an applied point force. It is puzzling that such a variety of signal-propagation mechanisms can be observed in different experiments and models. From a mathematical point of view, there are interesting related questions: If, for example, one wanted to model the silo illustrated in Figure 1, what boundary conditions should be specified? One hopes that the next generation of mathematical models—models that include the effects of large-scale fluctuations—will be able to answer these and related questions involving dense granular systems.

Together, the speakers in the two-part minisymposium on dense granular systems at ICIAM '07 provided an overview of various models and techniques. Robert Behr-inger opened the sessions with a discussion of jamming, plasticity, and diffusion. Subsequent speakers elaborated on jamming effects—Gregg Lois and Corey S. O'Hern, in cohesive systems, and David Head, in packings of aspherical beads.

Mattias Sperl discussed connections between granular systems and colloidal suspensions. Antoinette Tordesillas considered shear-banding from the point of view of bifurcation analysis within the frameworks of micromechanical continuum theory and discrete element simulations. Meenakshi Dutt focused on studies of dense granular systems relevant to the pharmaceutical industry. Closing the minisymposium, Lou Kondic discussed modeling the propagation of signals through dense granular systems.

*Lou Kondic is a professor of mathematics at the New Jersey Institute of Technology, Corey S. O'Hern is a professor in the Departments of Mechanical Engineering and Physics at Yale University, and Robert P. Behringer is a professor in the Department of Physics and director of the Center for Nonlinear and Complex Systems at Duke University.*

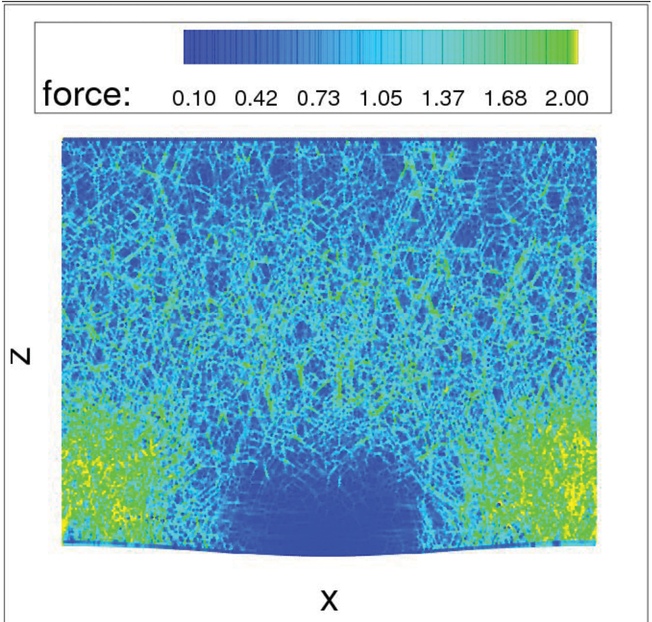


**Figure 2.** *Snapshot of force fluctuations occurring in discrete element simulations of a dense granular system. The system is perturbed by vibrating the lower boundary. (Kondic and Behringer, unpublished results.)*