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A Tale of Three Cities, Two Academies, and an *Opera Omnia* in 76 Volumes (and Counting)

Leonhard Euler. *By Emil A. Fellmann, translated from the German by Erika Gautschi and Walter Gautschi,* with illustrations, epilogue, notes, chronology, and bibliography, Birkhäuser, Basel, 2007, 179 pages, $39.95.

BOOK REVIEW

By Philip J. Davis

Entering a competition (on the optimal method for setting up a mast on a ship), Euler wrote, at age 19: "I did not find it necessary to confirm this theory of mine by experiment, because it is derived from the surest and most secure principles of mechanics."

Prior to reading Fellmann's splendid book, I knew only a tiny bit about Euler's mathematics and could have told you only three things about Euler the man (1707–1783): He was blind (initially in one eye, eventually in both), he fathered 13 children, and Catherine the Great of Russia treated him quite generously. I came upon these facts in my undergraduate reading of the short piece on Euler in Eric Temple Bell's *Men of Mathematics* (1937).

I now know much more. Emil A. Fellmann, a historian of science whose specialty is the 17th and 18th centuries, has chalked up many accomplishments and honors. Relevant to this review, he has been an active member of the Euler Commission of the Swiss Academy of Sciences (1972–1999) and the editor-in-chief of the 4th Series of Euler's *Opera Omnia* (1986–2006).

How is a biographer wallowing, indeed drowning, in a plethora of material to write an in-depth biography? Faced with such an impossibility, Fellmann has taken the right path, producing for the average informed reader a short biography (no formulas) that gets to the heart of Euler as a mathematician and as a human. Euler lived during the period of the great French Enlightenment. He hated its philosophy but ultimately, through his mathematics, contributed to the scientific enlightenment. I commend Fellmann for resisting the recent trend that has brought us so many 800-page biographies of back-breaking ponderousness.

Euler's life can be regarded as a Tale of Three Cities: Basel (for his first 20 years), Petersburg, Russia (the next 14 years), Berlin (25 years), and back to Petersburg (the last 17 years). It is also a Tale of Two Academies: in Petersburg and in Berlin. As regards the *Opera Omnia*, the third element in the title of this review, the publication of Euler's collected works began in 1907 and, a century later, has yet to be completed.

Why did Euler leave Basel for Petersburg in 1727? Simple answer: He couldn't find a job in his native Switzerland. Moreover, he would be getting in on the ground floor of the academy Peter (the Great) of Russia had set up in Petersburg in 1724. In the 18th century, it was in the academies that the research action was located. It was not in the universities, where, like as not, the emphasis was still on the classic curriculum and theology.

But after 13 years, Euler was ready to leave Petersburg, for a variety of reasons. First, in view of his large family and entourage, he feared the frequent fires that plagued the city. (His house burned down during his second stay.) There was also the very unpleasant and upsetting quartering of soldiers in private houses. (Cf. Amendment 3 to the U.S. Constitution: "No Soldier shall, in time of peace be quartered in any house, without the consent of the Owner, nor in time of war, but in a manner to be prescribed by law.") Also playing a role in his decision was the general Russification that had set in and was making life a bit uncomfortable for foreigners.

In 1740, unwilling to be outdone by his fellow monarch to the east, and hoping to revitalize the dormant and neglected Berlin–Brandenburg Society of Scientists started by Leibniz in 1700, Frederick II (the Great) of Prussia decided to set up an academy in Berlin and to populate it with the intellectual stars of the day. Frederick's personal knowledge and interest in mathematics amounted to zip, but someone probably told him that it would be classy to have a mathematician knocking about the place. Euler was on the short list, and in 1741 he arrived in Berlin. In 1746, Frederick wrote to his brother, Prince August Wilhelm:

> "I was afraid that your talk with Mr. Euler would not uplift you. . . . His epigrams consist of calculations of new curves, some sort of conic sections, or of astronomical measurements. Among the scholars there exist such monumental calculators . . . who are useful in the republic of the sciences, but are otherwise anything but brilliant. They are used as the Dorian columns in architecture. They belong to the subfloor, as support for the whole scientific edifice."

Twenty-five years later, why did Euler leave Berlin? Again, the decision was multiply determined. He never really hit it off with Frederick. Frederick loved the French intellectuals and the elegant free-thinking epigrammatic wits, and his academy and its wider circle were filled with these types. Being the devout son of a Calvinist parish pastor, Euler was a fish out of water. Moreover, Frederick, who for long periods of time was busy conquering Silesia, brushed off Euler's requests for the advancement of his children (only four of whom outlived their father). Bringing matters to a head, Catherine (the Great) of Russia, also on the prowl for stars, dangled a carrot in front of Euler in the form of 10,800 rubles of up-front money and a splendid salary. In 1766 Euler was back in the saddle in Petersburg (with the temporary loan of one of Catherine's cooks for his burgeoning household).

Fellmann has described in sufficient detail the haggling, the jealousies, the back-biting, the "who's going to run the show," the stealing of stars, that went on in the 18th-century world of academies. A reader might well ask, Have things changed at all in two and a half centuries?

It would be superfluous to remind the *SIAM News* readership that Euler was all over the map in pure and applied mathematics, physics, and astronomy. As a crude measure, if you go to the *CRC Concise Encyclopedia of Mathematics*, compiled by Eric Weisstein, you will find more than 70 entries under Euler. Gauss has 61, Hilbert 17, and Poincaré 13. Unless the pharmacologists and search engineers together find a remarkable mathematical brain enhancer, Euler's record will persist for quite a long time.

Fellmann makes special mention of Euler's work in mechanics, theory of ships, music theory, the calculus of variations, optics. In music, Euler stubbornly proposed a system for piano tuning (temperament) that "originated from a metaphysical–mathematical need" rather than from what musicians thought sounded good.

And among Euler's seemingly endless publications, *Introduction to the Analysis of the Infinite*, *New Principles of Gunnery* (i.e., ballistics), and *Dioptrics* receive special mention. The translators have included numerous clips from Euler's correspondence (in particular, his letters to Daniel Bernoulli), given in the original German orthography together with their English translations. Those who are able to handle the German can hear the true voice of the Master speaking.

Euler set forth some philosophic ideas that his contemporaries and later critics considered as so much "banality and absurdity." Paraphrasing the reaction of his friend Daniel Bernoulli, in a letter to Euler: You can't have it all, Leon. Stop messing around with the philosophic stuff. Shoemaker, stick to your last. Paradoxically and amusingly, two Basel mathematicians of more recent vintage expressed opposing views. Otto Spiess (whose 1929 book *Leonhard Euler* I surfed while preparing this review) agreed with Bernoulli. Andreas Speiser (1885–1970), on the other hand, thought that Euler was a precursor of modern philosophy through his influence on Kant.

As regards Euler's philosophy of mathematics, we can get an idea from his prize entry, at the age of 19, on the subject of the optimal method for setting up a mast on a ship (and this, Fellmann writes, from "a youthful inhabitant of the Alps who other than freighters, ferryboats, and simple canoes on the Rhine had never yet caught sight of a ship!"). Near the end of his entry, Euler wrote:

> "I did not find it necessary to confirm this theory of mine by experiment, because it is derived from the surest and most secure principles of mechanics, so that no doubt whatsoever can be raised on whether or not it be true and takes place in practice."

(The old jokes about the Swiss Navy were buried in 2002, when the *Alinghi*, the Swiss entry in the famous America's Cup yacht race, beat the New Zealanders in five straight. Alfio Quarteroni and his team at Ecole Polytechnique Fédérale de Lausanne, using hydro–aero–elasto software, served as consultants to the yacht designer as the Ghost of Euler nodded approvingly.)

Whether or not Euler believed all his life that beautiful math trumps experimentation, this view is still around today in many theoretical quarters. Fellman offers another bit of philosophy in a discussion of the great book on dioptrics:

> "Euler not only held onto the idea of the absence of chromatic error [refractivity varies with wavelength] in the eye, but even wanted to see therein a sure indication of the existence of God."

Given Euler's Calvinism, I suppose that he could be seen as a supporter of today's contentious "anthropic principle," which asserts that the fundamental physical constants of the universe have been so fine-tuned as to make you, me, and Euler distinct possibilities in the fullness of time.

What sort of a fellow was Euler? We get glimpses in Fellmann. "It appears that he was a caring father." Legend has him turning out paper after paper while dandling an infant on his knees. He was also demanding and stubborn in his opinions; a "meddler," according to his boss and scientific inferior at the Berlin academy, P.L.M. Maupertuis. (It was Voltaire who recommended Maupertuis to Frederick II.) After the death of his wife, Euler proposed to marry a widow of "low descent." His children were incensed: Who would get the property? He reneged (no breach-of-promise suit?) and some months later, at the age of 69, married the sister-in-law of his late wife. Thirty were present at the ceremony in his home, including grandchildren, all dolled up; speeches were made and a grand meal was enjoyed. But his children were not really reconciled to their new stepmother.

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Geniuses exist. There is no doubt of it, and Leonhard Euler was one of them. How do we ordinary folk react when confronted by the reality of genius? With awe, amazement, wonder, jealousy, anger; we confer notoriety, or arrange for prizes and monetary re-wards, encomia, testimonials, eulogies, festschrifts, collected works, marble monuments, and centenary celebrations. All are natural human impulses of long standing. Today, we also respond by supporting scientific inquiries into the cognitive and neural elements that contribute to genius, hoping some day to be able to create genius on demand.

Sometimes we play the "what if " game and receive some catharsis thereby. In Euler's case we look into his vast contributions to mathematics and wonder how the subsequent history of mathematics would have worked out if Euler had never been born. If you are an inevitabilist—like Tolstoy, who, at the end of *War and Peace*, wrote that the existence of Napoleon was irrelevant to the subsequent history of Europe—then you will believe that if mainstream mathematics has emerged from one person's work, it was inevitable that it emerged along the lines that it did. If you are in agreement with Thomas Carlyle and his Great Man theory of history, then you will assert that mathematics is nothing but the output of a few geniuses. If you are a social constructivist, you will believe that if other heads were around in other places, the subject of mathematics might have developed in a totally different way, leading to a non-equivalent corpus of material.

Fellmann ends his book with a collection of testimonials. The Eulerian age included such greats as Clairaut, d'Alembert, and Lagrange. Encyclopedist Denis Diderot, who knew a fair amount of mathematics, echoed a general contemporary testimonial, implying that when these fellows have passed on, mathematics will have come to an end. My testimony is simply that Euler was the Prince of Formalists. He paid no

attention to the questions of convergence or of continuity that would form the foundation of analysis, and yet was able to demonstrate the inherent power residing in the sheer manipulation of mathematical symbols.

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The year 2007 is the tercentenary of the birth of Leonhard Euler, and this event will be celebrated in Basel with a bang-up variety of activities extending over months. Information can be found at http://www.euler-2007.ch. The appearance of Erika and Walter Gautschi's translation of Fellmann's 1995 German-language biography, timed to coincide with the tercentenary, should be useful for a larger audience. I believe it is the only English-language biography of Euler of any substantial length.

On July 18, Walter Gautschi will present the Euler Lecture at the Sixth International Congress on Industrial and Applied Mathematics (Zürich, July 16–20).

Philip J. Davis, professor emeritus of applied mathematics at Brown University, is an independent writer, scholar, and lecturer. He lives in Providence, Rhode Island, and can be reached at philip_davis@brown.edu.