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Modeling Alcoholism as a Contagious Disease: How "Infected" Drinking Buddies Spread Problem Drinking

By Brandy Benedict

Do you drink?

Chances are, you do. According to a recent Gallup poll [2], 64% of American adults consider themselves drinkers, and about 20% think they sometimes drink too much. That ratio is even higher in some communities: At colleges and universities, where peer influence is especially strong, approximately one student in three abuses alcohol [4].

Do the drinking habits of friends affect a person's own level of alcohol consumption? Are you more likely to become an alcoholic if you spend a lot of time around alcoholics? How do drinking problems spread in a community of drinkers? Can you "catch" alcoholism?

A group of researchers are trying to answer these questions. At a standing-room-only talk at the 2006 Joint SMB–SIAM Conference on the Life Sciences in Raleigh, North Carolina, Fabio Sánchez of Cornell University presented a model for the spread of alcoholic drinking based on epidemiological models of infectious diseases [7]. The model, developed with collaborators Xiaohong Wang (Arizona State University), Carlos Castillo-Chávez (Arizona State), Dennis Gorman (Texas A&M Health Science Center), and Paul Gruenewald (Prevention Research Center, Berkeley, California), describes how alcoholics spread drinking problems through social contact between people with different drinking habits.

Social Factors in Modeling the Spread of Problem Drinking

To describe how alcoholism* spreads, and how reformed alcoholics relapse, the researchers divide the drinking population into three groups: susceptible drinkers (S), who consume alcohol in moderation but may one day develop problems with alcohol; alcoholics (A), who have drinking problems or addictions; and recovered individuals (R), former alcoholics who have entered treatment and are abstaining from alcohol. These are not the only possible classifications: "The distinction of drinking classes is not universal," says Sánchez, "and there's some debate as to what is a heavy drinker versus a moderate drinker or what is to be considered a drink." To simplify the model, the researchers place everyone who has a problem with alcohol, regardless of consumption level, in the same group.

People move from group to group as their drinking habits change. A moderate drinker can develop a drinking problem (move from S to A), an alcoholic can give up drinking (move from A to R), and a recovered alcoholic can relapse into alcoholism (move from R to A). A set of three nonlinear ordinary differential equations models the spread of drinking problems by describing changes in the populations of S, A, and R over time:

$$S' = \mu N - \beta \frac{A}{N} S - \mu S$$

$$A' = \beta \frac{S}{N} A + \rho \frac{R}{N} A - (\mu + \phi) A$$

$$R' = \phi A - \rho \frac{A}{N} R - \mu R.$$

Although the numbers of people in the groups change, Sánchez and his collaborators assume that the total population $N = S + A + R$ is constant, so that both birth (turning legal drinking age) and death occur at the same rate μ.

The researchers also assume that the rate at which alcoholism spreads depends on how often alcoholics meet people from other groups and how successful those encounters are in transmitting alcoholic habits. These social factors are embedded in the recruitment rate β and the relapse rate ρ, which represent the rates at which alcoholics convert susceptible drinkers and recovered alcoholics, respectively. The influence parameters β and ρ incorporate both the chance of an encounter and the likelihood of a conversion to problem drinking.

Along with the influence of alcoholics, the number of people who develop drinking problems depends also on the relative sizes of the populations, which determine the probability that two people from different groups meet. In this model no person is more popular than another, so that interactions between any two people are equally likely. This means that the probabilities of meeting a susceptible drinker, an alcoholic, or a recovered drinker at a bar are equal to the fractions of the groups in the total population: S/N, A/N, and R/N, respectively.

The rate at which an alcoholic "recruits" susceptible drinkers is proportional to the influence of alcoholics on susceptible drinkers (β) and the probability that an alcoholic meets a susceptible drinker (S/N). Because there are A alcoholics, the rate at which susceptible people develop drinking problems is $\beta A S/N$. In the same way, the rate at which (temporarily) recovered drinkers relapse and join the alcoholic group is $\rho A R/N$.

By adding the rates at which people enter the group and subtracting the rates at which people leave the group, we get an equation for the rate at which the population of the group changes over time. For example, the rate at which the number of alcoholics changes over time depends on

*The terms "alcoholism" and "alcoholic" are used in this article in the broadest possible sense to apply to individuals who have problems functioning in society due to their alcohol consumption, regardless of the frequency or quantity of their drinking.

the rates at which moderate drinkers become alcoholics ($\beta AS/N$), recovered alcoholics relapse ($\rho AR/N$), alcoholics give up alcohol and enter recovery (ϕA), and alcoholics die (μA):

$$A' = \beta \frac{S}{N} A + \rho \frac{R}{N} A - (\mu + \phi) A.$$

How Many Alcoholics Can One Alcoholic Make?

Analyzing these differential equations can give insight into how alcoholic drinking spreads and how the spread can be limited. An important tool for analyzing a contact model of this type is its reproductive number.

The reproductive number of the model is a dimensionless quantity that measures the vulnerability of a community to an epidemic of alcoholic drinking. It is computed in the idealized case in which the population is entirely susceptible or entirely recovered, and represents the number of people that a single alcoholic can convert to alcoholism.

Under further assumptions in a simpler model [1], the reproductive number is easily found and analyzed. If no recovered alcoholic ever relapses, and if the time interval is so short that there are no births or deaths, then the equations describing the rates of change in the populations of the groups are:

$$S' = -\beta \frac{S}{N} A$$

$$A' = \left(\beta \frac{S}{N} - \phi\right) A$$

$$R' = \phi A.$$

Because the number of people must be nonnegative ($S, A \geq 0$), and β is positive by definition, the population of susceptible drinkers can only decrease ($S' \leq 0$). The size of the alcoholic population remains constant when $\beta S/N - \phi = 0$, i.e., when $\beta S/(N\phi) = 1$. When the entire population is susceptible to infection ($S = N$), $\beta/\phi = 1$.

The quantity $R_0 = \beta/\phi$ is the reproductive number of this model. If $R_0 < 1$, the number of alcoholics in the community eventually drops to zero. When $R_0 > 1$, the number of alcoholics increases (because $A' > 0$), causing an epidemic of drinking in the community.

The more complex model for alcoholism developed by Sánchez and his colleagues incorporates relapse and has more than one reproductive number. In the absence of treatment (no R group), the basic reproductive number of the model is $R_0 = \beta/\mu$, where $1/\mu$ is the average time spent in a drinking environment.

Reproductive numbers can also be computed when treatment, and relapse, are possible. The relapse model, because it allows for people from two groups (S and R) to become alcoholic, has two reproductive numbers. The reproductive number with treatment, $R_\phi = \beta/(\mu + \phi)$, represents the number of susceptible drinkers converted to alcoholism under the influence of one alcoholic. If treatment is effective ($\phi > 0$), then $R_0 > R_\phi$. Fewer people will become alcoholic when effective treatment programs are in place. Relapse also affects the spread of alcoholism. The relapse reproductive number of the model, $R_\rho = \rho/(\mu + \phi)$, measures the number of recovered drinkers who revert to alcoholism under the direct influence of one alcoholic individual.

When the proportion of people becoming alcoholics over an interval of time is less than one, analysis of the simple model says that alcoholism should eventually disappear. For the more complicated model developed by Sánchez and his collaborators, however, this is not the case. Despite reproductive numbers less than one, a culture of drinking can flourish if relapse rates are high and treatment is not effective.

Equilibrium States

To determine the effects of reproductive numbers on the model dynamics, the researchers compute equilibrium states of the model. In solving for the equilibrium state, the three model equations can be reduced to one quadratic equation in A, with coefficients that depend on the reproductive numbers of the model. The equilibrium states are solutions to this quadratic equation.

Different equilibrium states can arise, depending on the value of the reproductive numbers with treatment. When both $R_\phi < 1$ and $R_\rho < 1$, the number of alcoholics will decrease to zero. When $R_\phi < 1$ and $R_\rho > 1$, an epidemic of alcoholism can arise if the initial proportion of alcoholics in the population is high.

By analyzing the reproductive numbers and equilibria of the differential equations, Sánchez and his collaborators are able to make important observations about alcoholism in communities. Their results indicate that ineffective treatment programs with high relapse rates may actually promote the spread of alcoholism, because they create a group of recovered drinkers who could easily relapse. An effective treatment strategy for curbing the spread of alcoholism may be to limit the amount of time a recovered alcoholic spends in places where drinking occurs.

The model presented by Sánchez differs from simpler epidemiological models in that the emergence of alcoholism depends also on the initial number of alcoholics. According to the group's numerical simulations, even if only 3% of the population are alcoholics, an epidemic of alcoholism is inevitable, regardless of the effectiveness of treatment programs.

While the model provides some insight into mechanisms for the spread of alcoholism and the effects of treatment programs on communities, incorporating addi-

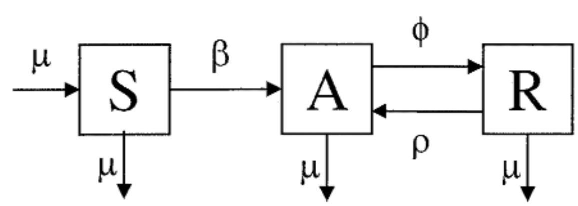

Figure 1. *A model for the spread of alcoholic drinking habits in a community of susceptible drinkers (S), alcoholics (A), and recovered drinkers (R). Arrows indicate the direction of movement into or out of a group, and parameters next to each arrow represent the rates governing each transition: recruitment rate (β), recovery rate (ϕ), relapse rate (ρ), birth/death rate (μ).*

tional features might make it possible to answer more questions. “This model addresses drinking behavior under some very general assumptions,” Sánchez says. “There are many details that could be included to make the model more realistic, but this is the first model of its kind and we wanted to understand the simplest case first, which turned out to have very rich dynamics.” Recovered drinkers could be allowed, for instance, to relapse into moderate drinking (S) rather than alcoholism (A). Researchers could also account for the geographic distance between people, and for varying levels of popularity, by modeling drinking and the spread of alcoholism with a small-world network, in which the chance of two people meeting is not necessarily the same for the entire population.

The contact model for the spread of alcoholism developed by Sánchez and his collaborators is not the only application of an epidemiological disease model. Models originally formulated to describe infectious diseases have also provided insights into other phenomena that depend on social contact and peer influence, such as crime [6], bulimia among college women [3], and the spread of rumors [5].

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