Summary of Forward Looking Panel Discussion, SIAM NW10

Peter D. Miller, University of Michigan, Ann Arbor (session chair)

September 6, 2010

The Forward Looking Panel Discussion was held in Philadelphia on Wednesday, August 18, 8:00-8:45 PM. The main idea of the session is contained in the corresponding abstract:

> This will be an open-ended discussion about the future of research in nonlinear waves and coherent structures. What are the big open problems, interesting new research directions, opportunities for collaboration, etc? After brief introductory remarks from the panelists, the chair will open the floor for questions from the audience.

The panelists for the session were:

- Miguel Alcubierre, Universidad Nacional Autónoma de México, Mexico
- William Kath, Northwestern University
- Govind Menon, Brown University
- Thomas Silva, National Institute of Standards and Technology
- Catherine Sulem, University of Toronto, Canada

Below follows a summary of each panelist's opening remarks, in the order in which they spoke. After the opening remarks, the floor was opened to questions. The chair recognized in particular H. Warchall, who asked the panelists whether they could comment on changes in the mathematical methodology appropriate for solving certain applied problems; Professor Warchall cited the increasing development of algorithms designed to work efficiently "with high probability" but not actually guaranteed to work at all. The chair also recognized P. Rosenau, who stressed the need to have methods and models that apply over a wide range of spatial and temporal scales; as a specific example Professor Rosenau pointed out how the continuum equations of fluid mechanics are inappropriate for modeling the flight of ultra high-altitude aircraft, but that it is hopelessly inefficient to deal with the more correct Boltzmann equation or other kinetic theories. These comments led to wide-ranging discussion among the panelists.

Opening Remarks of C. Sulem

Professor Sulem noted that there are good opportunities for future collaboration between people working in numerical methods and partial differential equations on the one hand, and people working in dynamical systems on the other hand. As historical evidence for this kind of interaction, she offered the development of symplectic numerical schemes and the corresponding theory of geometric integration. As a hot topic in this direction, Professor Sulem suggested the generalization of geometric integration methods to the realm of partial differential equations.

Opening Remarks of G. Menon

Professor Menon focused his remarks on the key role to be played by probability theory that he sees in the future of nonlinear waves research. As a specific example of how this is starting to occur, he cited the theory of rare events (large deviation theory) in the application of nonlinear fiber optics (error detection). He also pointed out how the theory of integrable systems, once strictly a subject of dynamical systems and nonlinear wave theory, has become very important in certain probabilistic problems. Here Professor Menon specifically mentioned the impact that asymptotic methods originally designed for integrable nonlinear partial differential equations describing nonlinear waves have had in successfully solving some longstanding problems in Random Matrix Theory and Combinatorics (Baik-Deift-Johansson theory). Professor Menon also made the observation that many more graduate students these days are comfortable with probability theory than in the past, and that this is a latent resource that should not be overlooked.

Opening Remarks of M. Alcubierre

Professor Alcubierre began his remarks by making the point that on the frontier of research in general relativity there are both open problems involving real physical applications as well as more theoretical applications. As an example of the latter he cited the problem of computing accurately high-speed collisions of black holes. He pointed out that this problem may become less theoretical with the Large Hadron Collider now online. As a potential source of physically interesting problems in general relativity, Professor Alcubierre reiterated the point made in his plenary talk that there are experiments now deployed throughout the world and hopefully soon also in solar orbit (the Laser Interferometer Space Antenna or LISA, a very detailed proposal of which has been favorably reviewed by both NASA and ESA and could perhaps be deployed as soon as 2020) that are designed to detect (for the first time) gravitational waves. He believes strongly that it is very likely that these waves will be detected soon, and that this will lead to a new urgency in modeling and numerically computing these waves.

As specific mathematical problems associated with numerical computation in general relativity, Professor Alcubierre stressed that we need to develop a better understanding of why current methods evidently work despite the presence of singularities in the gravitational field. He said that there is an urgent need for a theory to describe the proper handling of artificial boundary conditions that are required in numerical simulations. As many formulations of the fundamental equations of motion involve gauge choices that amount to viewing the dynamics as those of a dynamical system subject to certain constraints, there is a need for good theory of numerical methods for such constrained dynamical systems. In particular, Professor Alcubierre pointed out that in the numerical simulations currently in use the largest source of error comes from the numerical violation of physically consistent constraints. As a final remark on the role of gauge choices for field theories like general relativity, Professor Alcubierre posed the question of determining why some gauge choices (or formulations of the fundamental equations) are better than others in regard to their numerical implementation.

Opening Remarks of T. Silva

Professor Silva emphasized that the study of magnetization at the nanoscale is driven by real, current applications: the improvement of hard disks for computers, as well as the development of so-called magnetic RAM (random access memory) and so-called spin logic. He mentioned that the latter two applications are the targets of recent funding programs by DARPA. Professor Silva also said that the study of magnetic dynamics is also equally driven by applications, and here he cited telecommuncations and radar as relevant application areas. He summarized the current research panorama in magnetics by an equation:

$$\begin{aligned}\text{Lots of applications } + \text{ lots of materials advances } + \text{ lots of fabrication advances}\\ &= \text{lots of unknown physics}\\ &= \text{lots of opportunities for mathematicians.}\end{aligned}$$

Professor Silva observed that there are two distinct approaches that one can imagine in research on difficult problems in micromagnetics:

- The "old school" approach: reduce/simplify the problem until it can be handled by existing techniques.
- The "new school" approach: look for accurate approximations to solutions of the Landau-Lifschitz equation describing magnetization in physically and experimentally relevant geometries.

He then offered two specific examples to contrast these two approaches, and to stress that the "new school" approach is what is required, even though it may be a more difficult path.

Example: dealing with wave reflections from artificial boundary conditions in micromagnetics.

- "Old school" approach: put in some damping of the fields near the boundary. (The problem is that this still causes reflections, although they may be diminished.)
- "New school" approach: work out the correct absorbing boundary conditions for the Landau-Lifschitz model. This is an open problem.

Example: dealing with thermal fluctuations in continuous magnetic systems.

- "Old school" approach: extrapolate known results for magnetic particles to continuum magnetic systems. (The problem is that there is no basis whatsoever in theory for doing this!)
- "New school" approach: directly derive the correct stochastic terms for the partial differential equations of Landau-Lifschitz theory. This is again work yet to be done.

Opening Remarks of W. Kath

Professor Kath addressed his remarks to the audience focusing on what he believes we as researchers in nonlinear waves and coherent structures can do to remain actively involved in solving cutting-edge problems relevant in real applications. He maintained that our role as applied mathematicians could be to use methods of analysis to learn how to improve numerical schemes that are increasingly used and required to study complex problems, and also to learn how to effectively deal with random effects and the "curse of dimension" exhibited in large data sets.

As a practical suggestion, Professor Kath offered the advice of making the effort to talk to new people. He specifically pointed to the benefits of going to a professional meeting in an area outside one's own field as an eye-opening experience.