Abstract

Given a set Z of n positive integers and a target value t, the Subset Sum problem asks whether any subset of Z sums to t. A textbook pseudopolynomial time algorithm by Bellman from 1957 solves Subset Sum in time $O(nt)$. This has been improved to $O(n \max Z)$ by Pisinger [J. Algorithms'99] and recently to $\tilde{O}(\sqrt{n}t)$ by Koiliaris and Xu [SODA'17]. Here we present a simple and elegant randomized algorithm running in time $\tilde{O}(n+t)$. This improves upon a classic algorithm and is likely to be near-optimal, since it matches conditional lower bounds from Set Cover and k-Clique. We then use our new algorithm and additional tricks to improve the best known polynomial space solution from time $\tilde{O}(n^3 t)$ and space $\tilde{O}(n^2)$ to time $\tilde{O}(nt)$ and space $\tilde{O}(n \log t)$, assuming the Extended Riemann Hypothesis. Unconditionally, we obtain time $\tilde{O}(nt^{1+\varepsilon})$ and space $\tilde{O}(nt^{\varepsilon})$ for any constant $\varepsilon > 0$.