**Abstract**

A filtration over a simplicial complex $K$ is an ordering of the simplices of $K$ such that all prefixes in the ordering are subcomplexes of $K$. Filtrations are at the core of Persistent Homology, a major tool in Topological Data Analysis. In order to represent the filtration of a simplicial complex, the entire filtration can be appended to any data structure that explicitly stores all the simplices of the complex such as the Hasse diagram or the recently introduced Simplex Tree [Algorithmica '14]. However, with the popularity of various computational methods that need to handle simplicial complexes, and with the rapidly increasing size of the complexes, the task of finding a compact data structure that can still support efficient queries is of great interest. This direction has been recently pursued for the case of maintaining simplicial complexes. For instance, Boissonnat et al. [SoCG '15] considered storing the simplices that are maximal for the inclusion and Attali et al. [IJCGA '12] considered storing the simplices that block the expansion of the complex. Nevertheless, so far there has been no data structure that compactly stores the *filtration* of a simplicial complex, while also allowing the efficient implementation of basic operations on the complex. In this paper, we propose a new data structure called the Critical Simplex Diagram (CSD) which is a variant of the Simplex Array List (SAL) [SoCG '15]. Our data structure allows to store in a compact way the filtration of a simplicial complex, and allows for the efficient implementation of a large range of basic operations. Moreover, we prove that our data structure is essentially optimal with respect to the requisite storage space. Next, we show that the CSD representation admits the following construction algorithms.

- A new *edge-deletion* algorithm for the fast construction of Flag complexes, which only depends on the number of critical simplices and the number of vertices.

- A new *matrix-parsing* algorithm to quickly construct relaxed Delaunay complexes, depending only on the number of witnesses and the dimension of the complex.