Abstract

Let $G = (V, E)$ be a directed graph and $\ell\colon V \to [k] := \{1, \ldots, k\}$ a level assignment such that $\ell(u) < \ell(v)$ for all directed edges $(u, v) \in E$. A level planar drawing of G is a drawing of G where each vertex v is mapped to a unique point on the horizontal line ℓ_i with y-coordinate $\ell(v)$, and each edge is drawn as a y-monotone curve between its endpoints such that no two curves cross in their interior. In the problem Constrained Level Planarity (CLP for short), we are further given a partial ordering $\prec_i$ of $V_i := \ell^{-1}(i)$ for $i \in [k]$, and we seek a level planar drawing where the order of the vertices on ℓ_i is a linear extension of $\prec_i$. A special case of this is the problem Partial Level Planarity (PLP for short), where we are asked to extend a given level-planar drawing $\mathcal{H}$ of a subgraph $H \subseteq G$ to a complete drawing $\mathcal{G}$ of G without modifying the given drawing, i.e., the restriction of $\mathcal{G}$ to H must coincide with $\mathcal{H}$. We give a simple polynomial-time algorithm with running time $\mathcal{O}(n^5)$ for CLP of single-source graphs that is based on a simplified version of an existing level-planarity testing algorithm for single-source graphs. We introduce a modified type of PQ-tree data structure that is capable of efficiently handling the arising constraints to improve the running time to $O(n + k\ell)$, where ℓ is the size of the constraints. We complement this result by showing that PLP is NP-complete even in very restricted cases. In particular, PLP remains NP-complete even when G is a subdivision of a triconnected planar graph with bounded degree.