Abstract

In this paper, we study a set of combinatorial optimization problems on weighted graphs: the shortest path problem with negative weights, the weighted perfect bipartite matching problem, the unit-capacity minimum-cost maximum flow problem, and the weighted perfect bipartite b-matching problem under the assumption that $\|b\|_1 = O(m)$. We show that each of these four problems can be solved in $\tilde{O}(m^{10/7} \log W)$ time, where W is the absolute maximum weight of an edge in the graph, providing the first polynomial improvement in their sparse-graph time complexity in over 25 years.