Abstract

We consider the problem of estimating the value of MAX-CUT in a graph in the streaming model of computation. We show that there exists a constant $\epsilon_* > 0$ such that any randomized streaming algorithm that computes a $(1 + \epsilon_*)$-approximation to MAX-CUT requires $\Omega(n)$ space on an n vertex graph. By contrast, there are algorithms that produce a $(1 + \epsilon)$-approximation in space $O(n/\epsilon^2)$ for every $\epsilon > 0$. Our result is the first linear space lower bound for the task of approximating the max cut value and partially answers an open question from the literature. The prior state of the art ruled out $(2 - \epsilon)$-approximation in $\tilde{O}(\sqrt{n})$ space or $(1 + \epsilon)$-approximation in $n^{1-O(\epsilon)}$ space, for any $\epsilon > 0$. Previous lower bounds for the MAX-CUT problem relied, in essence, on a lower bound on the communication complexity of the following task: Several players are each given some edges of a graph and they wish to determine if the union of these edges is ϵ-close to forming a bipartite graph, using one-way communication. The previous works proved a lower bound of $\Omega(\sqrt{n})$ for this task when $\epsilon = 1/2$, and $n^{1-O(\epsilon)}$ for every $\epsilon > 0$, even when one of the players is given a candidate bipartition of the graph and the graph is promised to be bipartite with respect to this partition or ϵ-far from bipartite. This added information was essential in enabling the previous analyses but also yields a weak bound since, with this extra information, there is an $n^{1-O(\epsilon)}$ communication protocol for this problem. In this work, we give an $\Omega(n)$ lower bound on the communication complexity of the original problem (without the extra information) for $\epsilon = \Omega(1)$ in the three-player setting. Obtaining this $\Omega(n)$ lower bound on the communication complexity is the main technical result in this paper. We achieve it by a delicate choice of distributions on instances as well as a novel use of the convolution theorem from Fourier analysis combined with graph-theoretic considerations to analyze the communication complexity.