Abstract

Stochastic convex optimization, where the objective is the expectation of a random convex function, is an important and widely used method with numerous applications in machine learning, statistics, operations research and other areas. We study the complexity of stochastic convex optimization given only *statistical query* (SQ) access to the objective function. We show that well-known and popular first-order iterative methods can be implemented using only statistical queries. For many cases of interest we derive nearly matching upper and lower bounds on the estimation (sample) complexity including linear optimization in the most general setting. We then present several consequences for machine learning, differential privacy and proving concrete lower bounds on the power of convex optimization based methods. The key ingredient of our work is SQ algorithms and lower bounds for estimating the mean vector of a distribution over vectors supported on a convex body in $\mathbb{R}^d$. This natural problem has not been previously studied and we show that our solutions can be used to get substantially improved SQ versions of Perceptron and other online algorithms for learning halfspaces.