Abstract

Analyzing the mixing time of random walks is a well-studied problem with applications in random sampling and more recently in graph partitioning. In this work, we present new analysis of random walks and evolving sets using more combinatorial graph structures, and show some implications in approximating small-set expansion. On the other hand, we provide examples showing the limitations of using random walks and evolving sets in disproving the small-set expansion hypothesis.

1. We define a combinatorial analog of the spectral gap, and use it to prove the convergence of non-lazy random walks. A corollary is a tight lower bound on the small-set expansion of graph powers for any graph.
2. We prove that random walks converge faster when the robust vertex expansion of the graph is larger. This provides an improved analysis of the local graph partitioning algorithm using the evolving set process, and also derives an alternative proof of an improved Cheeger's inequality.
3. We give an example showing that the evolving set process fails to disprove the small-set expansion hypothesis. This refutes a conjecture of Oveis Gharan and shows the limitations of all existing local graph partitioning algorithms in approximating small-set expansion.