Abstract

Connectivity related concepts are of fundamental interest in graph theory. The area has received extensive attention over four decades, but many problems remain unsolved, especially for directed graphs. A directed graph is 2-edge-connected (resp., 2-vertex-connected) if the removal of any edge (resp., vertex) leaves the graph strongly connected. In this paper we present improved algorithms for computing the maximal 2-edge- and 2-vertex-connected subgraphs of a given directed graph. These problems were first studied more than 35 years ago, with $\widetilde{O}(mn)$ time algorithms for graphs with m edges and n vertices being known since the late 1980s. In contrast, the same problems for undirected graphs are known to be solvable in linear time. Henzinger et al. [ICALP 2015] recently introduced $O(n^2)$ time algorithms for the directed case, thus improving the running times for dense graphs. Our new algorithms run in time $O(m^{3/2})$, which further improves the running times for sparse graphs. The notion of 2-connectivity naturally generalizes to k-connectivity for $k > 2$. For constant values of k, we extend one of our algorithms to compute the maximal k-edge-connected in time $O(m^{3/2} \log n)$, improving again for sparse graphs the best known algorithm by Henzinger et al. [ICALP 2015] that runs in $O(n^2 \log n)$ time.