

Abstract

We investigate a new class of congestion games, called *Totally Unimodular (TU) Congestion Games*, where the players' strategies are binary vectors inside polyhedra defined by totally unimodular constraint matrices. Network congestion games belong to this class. In the symmetric case, when all players have the same strategy set, we design an algorithm that finds an optimal aggregated strategy and then decomposes it into the single players' strategies. This approach yields strongly polynomial-time algorithms to (i) find a pure Nash equilibrium, and (ii) compute a socially optimal state, if the delay functions are weakly convex. We also show how this technique can be extended to matroid congestion games. We then introduce some combinatorial TU congestion games, where the players' strategies are matchings, vertex covers, edge covers, and stable sets of a given bipartite graph. In the asymmetric case, we show that for these games (i) it is PLS-complete to find a pure Nash equilibrium even in case of linear delay functions, and (ii) it is NP-hard to compute a socially optimal state, even in case of weakly convex delay functions.