Abstract

Permutation Pattern Matching (or PPM) is a decision problem whose input is a pair of permutations π and τ, represented as sequences of integers, and the task is to determine whether τ contains a subsequence order-isomorphic to π. Bose, Buss and Lubiw proved that PPM is NP-complete on general inputs. We show that PPM is NP-complete even when π has no decreasing subsequence of length 3 and τ has no decreasing subsequence of length 4. This provides the first known example of PPM being hard when one or both of π and τ are restricted to a proper hereditary class of permutations. This hardness result is tight in the sense that PPM is known to be polynomial when both π and τ avoid a decreasing subsequence of length 3, as well as when π avoids a decreasing subsequence of length 2. The result is also tight in another sense: we will show that for any hereditary proper subclass $\mathcal{C}$ of the class of permutations avoiding a decreasing sequence of length 3, there is a polynomial algorithm solving PPM instances where π is from $\mathcal{C}$ and τ is arbitrary. We also obtain analogous hardness and tractability results for the class of so-called skew-merged patterns. From these results, we deduce a complexity dichotomy for the PPM problem restricted to π belonging to $\mathrm{Av}(\alpha)$, where $\mathrm{Av}(\alpha)$ denotes the class of permutations avoiding a permutation α. Specifically, we show that the problem is polynomial when α is in the set $\{1, 12, 21, 132, 213, 231, 312\}$, and it is NP-complete for any other α.