Abstract

We give a combinatorial condition for the existence of efficient, LP-based FPT algorithms for a broad class of graph-theoretical optimisation problems. Our condition is based on the notion of biased graphs known from matroid theory. Specifically, we show that given a biased graph $\Psi = (G, \mathcal{B})$, where $\mathcal{B}$ is a class of balanced cycles in G, the problem of finding a set X of at most k vertices in G which intersects every unbalanced cycle in G admits an FPT algorithm using an LP-branching approach, similar to those previously seen for VCSP problems (Wahlström, SODA 2014). Our algorithm has two parts. First we define a *local problem*, where we are additionally given a root vertex $v_0 \in V$ and asked only to delete vertices X (excluding v_0) so that the connected component of v_0 in $G - X$ contains no unbalanced cycle. We show that this local problem admits a persistent, half-integral LP-relaxation with a polynomial-time solvable separation oracle, and can therefore be solved in FPT time via LP-branching, assuming only oracle membership queries for the class of balanced cycles in G. We then show that solutions to this local problem can be used to tile the graph, producing an optimal solution to the original, global problem as well. This framework captures many of the problems previously solved via the VCSP approach to LP-branching, as well as new generalisations, such as Group Feedback Vertex Set for infinite groups (e.g., for graphs whose edges are labelled by matrices). A major advantage compared to previous work is that it is immediate to check the applicability of the result for a given problem, whereas testing applicability of the VCSP approach for a specific VCSP, requires determining the existence of an embedding language with certain algebraically defined properties, which is not known to be decidable in general.