

Abstract

Nonnegative matrix factorization (NMF) is the problem of decomposing a given nonnegative $n \times m$ matrix M into a product of a nonnegative $n \times d$ matrix W and a nonnegative $d \times m$ matrix H . NMF has a wide variety of applications, including bioinformatics, chemometrics, communication complexity, machine learning, polyhedral combinatorics, among many others. A longstanding open question, posed by Cohen and Rothblum in 1993, is whether every rational matrix M has an NMF with minimal d whose factors W and H are also rational. We answer this question negatively, by exhibiting a matrix M for which W and H require irrational entries. As an application of this result, we show that state minimization of labeled Markov chains can require the introduction of irrational transition probabilities. We complement these irrationality results with an NP-complete version of NMF for which rational numbers suffice.