Abstract

A *binary matrix* is a matrix with entries from the set $\{0, 1\}$. We say that a binary matrix A *contains* a binary matrix S if S can be obtained from A by removal of some rows, some columns, and changing some 1-entries to 0-entries. If A does not contain S, we say that A *avoids* S. A k-*permutation matrix* P is a binary $k \times k$ matrix with exactly one 1-entry in every row and one 1-entry in every column. The Füredi–Hajnal conjecture, proved by Marcus and Tardos, states that for every permutation matrix P, there is a constant c_P such that for every $n \in \mathbb{N}$, every $n \times n$ binary matrix A with at least $c_P n$ 1-entries contains P. We show that $c_P \leq 2^{O(k^{2/3} \log^{7/3} k / (\log \log k)^{1/3})}$ asymptotically almost surely for a random k-permutation matrix P. We also show that $c_P \leq 2^{(4+o(1))k}$ for every k-permutation matrix P, improving the constant in the exponent of a recent upper bound on c_P by Fox. We also consider a higher-dimensional generalization of the Stanley–Wilf conjecture about the number of d-dimensional n-permutation matrices avoiding a fixed d-dimensional k-permutation matrix, and prove almost matching upper and lower bounds of the form $(2^k)^{O(n)} \cdot (n!)^{d-1-1/(d-1)}$ and $n^{-O(k)} k^{\Omega(n)} \cdot (n!)^{d-1-1/(d-1)}$, respectively.