Abstract

Nowhere dense classes of graphs (J. Nešetřil and P. Ossona de Mendez, *First order properties on nowhere dense structures*, The Journal of Symbolic Logic, Vol. 75(03), pages 868–887, 2010.), (J. Nešetřil and P. Ossona de Mendez, *On nowhere dense graphs*, European Journal of Combinatorics, Vol. 32(4), pages 600–617, 2011.) are very general classes of uniformly sparse graphs with several seemingly unrelated characterisations. From an algorithmic perspective, a characterisation of these classes in terms of *uniform quasi-wideness*, a concept originating in finite model theory, has proved to be particularly useful. Uniform quasi-wideness is used in many fpt-algorithms on nowhere dense classes. However, the existing constructions showing the equivalence of nowhere denseness and uniform quasi-wideness imply a non-elementary blow up in the parameter dependence of the fpt-algorithms, making them infeasible in practice. As a first main result of this paper, we use tools from logic, in particular from a sub-field of model theory known as stability theory, to establish polynomial bounds for the equivalence of nowhere denseness and uniform quasi-wideness. As an algorithmic application of our new methods, we obtain for every fixed value of $r \in \mathbb{N}$ a polynomial kernel for the distance-r dominating set problem on nowhere dense classes of graphs. This is particularly interesting, as it implies that for every subgraph-closed class $\mathcal{C}$, the distance-r dominating set problem admits a kernel on $\mathcal{C}$ for every value of r if, and only if, it admits a polynomial kernel for every value of r (under the standard assumption of parameterized complexity theory that $\mathrm{FPT} \neq \mathrm{W}[2]$). Finally, we demonstrate how to use the new methods to improve the parameter dependence of many fixed-parameter algorithms. As an example we provide a single exponential parameterized algorithm for the Connected Dominating Set problem on nowhere dense graph classes.