## Abstract

Let $C_1, ..., C_{d+1}$ be $d+1$ point sets in $\mathbb{R}^d$, each containing the origin in its convex hull. A subset $C$ of $\bigcup_{i=1}^{d+1} C_i$ is called a colorful choice (or rainbow) for $C_1, \ldots, C_{d+1}$, if it contains exactly one point from each set $C_i$. The colorful Caratheodory theorem states that there always exists a colorful choice for $C_1, \ldots, C_{d+1}$ that has the origin in its convex hull. This theorem is very general and can be used to prove several other existence theorems in high-dimensional discrete geometry, such as the centerpoint theorem or Tverberg's theorem. The colorful Caratheodory problem (CCP) is the computational problem of finding such a colorful choice. Despite several efforts in the past, the computational complexity of CCP in arbitrary dimension is still open. We show that CCP lies in the intersection of the complexity classes PPAD and PLS. This makes it one of the few geometric problems in PPAD and PLS that are not known to be solvable in polynomial time. Moreover, it implies that the problem of computing centerpoints, computing Tverberg partitions, and computing points with large simplicial depth is contained in PPAD $\cap$ PLS. This is the first nontrivial upper bound on the complexity of these problems. Finally, we show that our PPAD formulation leads to a polynomial-time algorithm for a special case of CCP in which we have only two color classes $C_1$ and $C_2$ in $d$ dimensions, each with the origin in its convex hull, and we would like to find a set with half the points from each color class that contains the origin in its convex hull.