Abstract

We give a sharp lower bound on the number of matchings of a given size in a bipartite graph. When specialized to regular bipartite graphs, our results imply Schrijver's theorem and Friedland's Lower Matching Conjecture proven by Gurvits and Csikvári. Indeed, our work extends the recent work of Csikvári done for regular and bi-regular bipartite graphs. Moreover, our lower bounds are order optimal as they are attained for a sequence of 2-lifts of the original graph as well as for random n-lifts of the original graph when n tends to infinity. We then extend our results to permanents and subpermanents sums. For permanents, we are able to recover the lower bound of Schrijver recently proved by Gurvits using stable polynomials. Our proof is algorithmic and borrows ideas from the theory of local weak convergence of graphs, statistical physics and covers of graphs. We provide new lower bounds for subpermanents sums and obtain new results on the number of matchings in random n-lifts with some implications for the matching measure and the spectral measure of random n-lifts as well as for the spectral measure of infinite trees.