

Abstract

Local search proved to be an extremely useful tool when facing hard optimization problems (e.g., via the simplex algorithm, simulated annealing, or genetic algorithms). Although powerful, it has its limitations: there are functions for which exponentially many queries are needed to find a local optimum. In many contexts the optimization problem is defined by a continuous function, which might offer an advantage when performing the local search. This leads us to study the following natural question: How hard is *continuous* local search? The computational complexity of such search problems is captured by the complexity class **CLS** (Daskalakis and Papadimitriou SODA'11) which is contained in the intersection of **PLS** and **PPAD**, two important subclasses of **TFNP** (the class of **NP** search problems with a guaranteed solution). In this work, we show the first hardness results for **CLS** (the smallest non-trivial class among the currently defined subclasses of **TFNP**). Our hardness results are in terms of black-box (where only oracle access to the function is given) and white-box (where the function is represented succinctly by a circuit). In the black-box case, we show instances for which any (computationally unbounded) randomized algorithm must perform exponentially many queries in order to find a local optimum. In the white-box case, we show hardness for computationally bounded algorithms under cryptographic assumptions. Our results demonstrate a strong conceptual barrier precluding design of efficient algorithms for solving local search problems even over continuous domains. As our main technical contribution we introduce a new total search problem which we call **END-OF-METERED-LINE**. The special structure of **END-OF-METERED-LINE** enables us to: (1) show that it is contained in **CLS**, and (2) prove hardness for it both in the black-box and the white-box setting.