**Abstract**

We consider the online traveling salesperson problem (TSP), where requests appear online over time on the real line and need to be visited by a server initially located at the origin. We distinguish between closed and open online TSP, depending on whether the server eventually needs to return to the origin or not. While online TSP on the line is a very natural online problem that was introduced more than two decades ago, no tight competitive analysis was known to date. We settle this problem by providing tight bounds on the competitive ratios for both the closed and the open variant of the problem. In particular, for closed online TSP, we provide a 1.64-competitive algorithm, thus matching a known lower bound. For open online TSP, we give a new upper bound as well as a matching lower bound that establish the remarkable competitive ratio of 2.04.

Additionally, we consider the online DIAL-A-RIDE problem on the line, where each request needs to be transported to a specified destination. We provide an improved non-preemptive lower bound of 1.75 for this setting, as well as an improved preemptive algorithm with competitive ratio 2.41.

Finally, we generalize known and give new complexity results for the underlying offline problems. In particular, we give an algorithm with running time $O(n^2)$ for closed offline TSP on the line with release dates and show that both variants of offline DIAL-A-RIDE on the line are NP-hard for any capacity $c \geq 2$ of the server.