Abstract

We show that for any constant $\Delta \geq 2$, there exists a graph Γ with $O(n^{\Delta/2})$ vertices which contains every n-vertex graph with maximum degree Δ as an induced subgraph. For odd Δ this significantly improves the best-known earlier bound of Esperet et al. and is optimal up to a constant factor, as it is known that any such graph must have at least $\Omega(n^{\Delta/2})$ vertices. Our proof builds on the approach of Alon and Capalbo (SODA 2008) together with several additional ingredients. The construction of Γ is explicit and is based on an appropriately defined composition of high-girth expander graphs. The proof also provides an efficient deterministic procedure for finding, for any given input graph H on n vertices with maximum degree at most Δ, an induced subgraph of Γ isomorphic to H.