Abstract

In the classic *circle packing problem*, one asks whether a given set of circles can be packed into the unit square. This problem is known to be NP-hard. In this paper, we present a new sufficient condition using only the circles' combined area: It is possible to pack any circle instance with a combined area of up to ≈ 0.5390. This bound is tight, in the sense that for any larger combined area, there are instances which cannot be packed, which is why we call this number the problem's *critical density*. Similar results have long been known for squares, but to the best of our knowledge, this paper gives the first results of this type for circular objects. Our proof is constructive: We describe a subdivision scheme which recursively splits the circles into groups and then packs these into subcontainers. We call this algorithm *Split Packing*. Beside realizing all packings up to the critical density bound, Split Packing also serves as a constant-factor approximation algorithm when looking for the smallest square in which a given set of circles can be packed. We believe that the ideas behind Split Packing are interesting and elegant on their own, and we see many opportunities to apply this technique in the context of other packing and covering problems. A browser-based, interactive visualization of the Split Packing approach and other related material can be found at https://morr.cc/split-packing/.