

## Abstract

We study the non-uniform capacitated multi-item lot-sizing (CMILS) problem. In this problem, there is a set of demands over a planning horizon of  $T$  time periods and all demands must be satisfied on time. We can place an order at the beginning of each period  $s$ , incurring an ordering cost  $K_s$ . The total quantity of all products ordered at time  $s$  can not exceed a given capacity  $C_s$ . On the other hand, carrying inventory from time to time incurs inventory holding cost. The goal of the problem is to find a feasible solution that minimizes the sum of ordering and holding costs. Levi et al. (Levi, Lodi and Sviridenko, *Mathematics of Operations Research* 33(2), 2008) gave a 2-approximation for the problem when the capacities  $C_s$  are the same. In this paper, we extend their result to the case of non-uniform capacities. That is, we give a constant approximation algorithm for the capacitated multi-item lot-sizing problem with general capacities. The constant approximation is achieved by adding an exponentially large set of new covering inequalities to the natural facility-location type linear programming relaxation for the problem. Along the way of our algorithm, we reduce the CMILS problem to two generalizations of the classic knapsack covering problem. We give LP-based constant approximation algorithms for both generalizations, via the iterative rounding technique.