Abstract

We consider the following streaming problem: given a hardwired $m \times n$ matrix A together with a poly(mn)-bit hardwired string of advice that may depend on A, for any x with coordinates $x_1, \ldots x_n$ presented in order, output the coordinates of $A \cdot x$ in order. Our focus is on using as little memory as possible while computing $A \cdot x$; we do not count the size of the output tape on which the coordinates of $A \cdot x$ are written; for some matrices A such as the identity matrix, a constant number of words of space is achievable. Such an algorithm has to adapt its memory contents to the changing structure of A and exploit it on the fly. We give a nearly tight characterization, for any number of passes over the coordinates of x, of the space complexity of such a streaming algorithm. Our characterization is constructive, in that we provide an efficient algorithm matching our lower bound on the space complexity. The essential parameters, *streaming rank* and *multi-pass streaming rank* of A, might be of independent interest, and we show they can be computed efficiently. We give several applications of our results to computing Johnson-Lindenstrauss transforms. Finally, we note that we can characterize the optimal space complexity when the coordinates of $A \cdot x$ can be written in any order.