Abstract

We introduce new data structures for answering connectivity queries in graphs subject to batched *vertex failures*. Our deterministic structure processes a batch of $d \leq d_*$ failed vertices in $\tilde{O}(d^3)$ time and thereafter answers connectivity queries in $O(d)$ time. It occupies space $O(d_* m \log n)$. We develop a randomized Monte Carlo version of our data structure with update time $\tilde{O}(d^2)$, query time $O(d)$, and space $\tilde{O}(m)$ for any d_*. This is the first connectivity oracle for general graphs that can efficiently deal with an unbounded number of vertex failures. Our data structures are based on a new decomposition theorem for an undirected graph $G = (V, E)$, which is of independent interest. It states that for any terminal set $U \subseteq V$ we can remove a set B of $|U|/(s-2)$ vertices such that the remaining graph contains a Steiner forest for $U - B$ with maximum degree s.