Abstract

For a simple (unbiased) random walk on a connected graph with n vertices, the cover time (the expected number of steps it takes to visit all vertices) is at most $O(n^3)$. We consider locally biased random walks, in which the probability of traversing an edge depends on the degrees of its endpoints. We confirm a conjecture of Abdullah, Cooper and Draief [2015] that the min-degree local bias rule ensures a cover time of $O(n^2)$. For this we formulate and prove the following lemma about spanning trees. Let $R(e)$ denote for edge e the minimum degree among its two endpoints. We say that a weight function W for the edges is feasible if it is nonnegative, dominated by R (for every edge $W(e) \leq R(e)$) and the sum over all edges of the ratios $W(e)/R(e)$ equals $n-1$. For example, in trees $W(e) = R(e)$, and in regular graphs the sum of edge weights is $d(n-1)$. **Lemma:** for every feasible W, the minimum weight spanning tree has total weight $O(n)$. For regular graphs, a similar lemma was proved by Kahn, Linial, Nisan and Saks [1989].