Abstract

We revisit the classic problem of dynamically maintaining shortest paths between all pairs of nodes of a directed weighted graph. The allowed updates are insertions and deletions of nodes and their incident edges. We give worst-case guarantees on the time needed to process a single update (in contrast to related results, the update time is *not* amortized over a sequence of updates). Our main result is a simple randomized algorithm that for any parameter $c > 1$ has a worst-case update time of $O(cn^{2+2/3} \log^{4/3} n)$ and answers distance queries correctly with probability $1 - 1/n^c$, against an adaptive online adversary if the graph contains no negative cycle. The best deterministic algorithm is by Thorup [STOC 2005] with a worst-case update time of $\tilde{O}(n^{2+3/4})$ and assumes non-negative weights. This is the first improvement for this problem for more than a decade. Conceptually, our algorithm shows that randomization along with a more direct approach can provide better bounds.