

Abstract

We prove that any mixed-integer linear extended formulation for the matching polytope of the complete graph on n vertices, with a polynomial number of constraints, requires $\Omega(\sqrt{\frac{n}{\log n}})$ many integer variables. By known reductions, this result extends to the traveling salesman polytope. This lower bound has various implications regarding the existence of small mixed-integer mathematical formulations of common problems in operations research. In particular, it shows that for many classic vehicle routing problems and problems involving matchings, any compact mixed-integer linear description of such a problem requires a large number of integer variables. This provides a first non-trivial lower bound on the number of integer variables needed in such settings.