Abstract

We study the problem of finding a tour of n points in which *every edge* is *long*. More precisely, we wish to find a tour that visits every point exactly once, maximizing the length of the shortest edge in the tour. The problem is known as Maximum Scatter TSP, and was introduced by Arkin et al. (SODA 1997), motivated by applications in manufacturing and medical imaging. Arkin et al. gave a 0.5-approximation for the metric version of the problem and showed that this is the best possible ratio achievable in polynomial time (assuming P $\neq$ NP). Arkin et al. raised the question of whether a better approximation ratio can be obtained in the Euclidean plane. We answer this question in the affirmative in a more general setting, by giving a $(1-\epsilon)$-approximation algorithm for d-dimensional doubling metrics, with running time $\tilde{O}\left(n^3 + 2^{O(K \log K)}\right)$, where $K \leq \left(\frac{13}{\epsilon}\right)^d$. As a corollary we obtain (i) an efficient polynomial-time approximation scheme (EPTAS) for all constant dimensions d, (ii) a polynomial-time approximation scheme (PTAS) for dimension $d = \log\log n/c$, for a sufficiently large constant c, and (iii) a PTAS for constant d and $\epsilon = \Omega(1/\log\log n)$. Furthermore, we show the dependence on d in our approximation scheme to be essentially optimal, unless Satisfiability can be solved in subexponential time.