Abstract

We study a family of closely-related distributed graph problems, which we call *degree splitting*, where roughly speaking the objective is to partition (or orient) the edges such that each node's degree is split almost uniformly. Our findings lead to answers for a number of problems, a sampling of which includes: (1) We present a poly$\log n$ round deterministic algorithm for $(2\Delta-1)\cdot(1+o(1))$ *edge-coloring*, where Δ denotes the maximum degree. Modulo the $1+o(1)$ factor, this settles one of the long-standing open problems of the area from the 1990's (see e.g. Panconesi and Srinivasan [PODC'92]). Indeed, a weaker requirement of $(2\Delta-1)\cdot\text{polylog}(\Delta)$ edge-coloring in $\text{polylog}\, n$ rounds was asked for in the 4th open question in the *Distributed Graph Coloring* book by Barenboim and Elkin. (2) We show that *sinkless orientation*—i.e., orienting edges such that each node has at least one outgoing edge—on Δ-regular graphs can be solved in $O(\log_\Delta \log n)$ rounds randomized and in $O(\log_\Delta n)$ rounds deterministically. These prove the corresponding lower bounds by Brandt et al. [STOC'16] and Chang, Kopelowitz, and Pettie [FOCS'16] to be tight. Moreover, these show that sinkless orientation exhibits an exponential separation between its randomized and deterministic complexities, akin to the results of Chang et al. for Δ-coloring Δ-regular trees. (3) We present a randomized $O(\log^4 n)$ round algorithm for orienting a-arboricity graphs with maximum out-degree $a(1+\epsilon)$. This can be also turned into a decomposition into $a(1+\epsilon)$ forests when $a=\Omega(\log n)$ and into $a(1+\epsilon)$ pseduo-forests when $a=o(\log n)$. Obtaining an efficient distributed decomposition into less than $2a$ forests was stated as the 10th open problem in the book by Barenboim and Elkin.