Abstract

For any integer $n \geq 1$ a *middle levels Gray code* is a cyclic listing of all n-element and $(n+1)$-element subsets of $\{1, 2, \ldots, 2n+1\}$ such that any two consecutive subsets differ in adding or removing a single element. The question whether such a Gray code exists for any $n \geq 1$ has been the subject of intensive research during the last 30 years, and has been answered affirmatively only recently [T. Mütze. Proof of the middle levels conjecture. *Proc. London Math. Soc.*, 112(4):677–713, 2016]. In a follow-up paper [T. Mütze and J. Nummenpalo. An efficient algorithm for computing a middle levels Gray code. *Proc. ESA*, 2015] this existence proof was turned into an algorithm that computes each new set in the Gray code in time $\mathcal{O}(n)$ on average. In this work we complete this line of research by presenting an algorithm for computing a middle levels Gray code in optimal time and space: Each new set is generated in time $\mathcal{O}(1)$, and the required space is $\mathcal{O}(n)$.