Abstract

There has been significant success in designing highly efficient algorithms for distance and shortest-path queries in recent years; many of the state-of-the-art algorithms use the hub labeling framework. In this paper, we study the approximability of the Hub Labeling problem. We prove a hardness of $\Omega(\log n)$ for Hub Labeling, matching known approximation guarantees. The hardness result applies to graphs that have multiple shortest paths between some pairs of vertices. No hardness of approximation results were known previously. Then, we focus on graphs that have a unique shortest path between each pair of vertices. This is a very natural family of graphs, and much research on the Hub Labeling problem has studied such graphs. We give an $O(\log D)$ approximation algorithm for graphs of shortest-path diameter D with unique shortest paths. In particular, we get an $O(\log \log n)$ approximation for graphs of polylogarithmic diameter, while previously known algorithms gave an $O(\log n)$ approximation. Finally, we present a polynomial-time approximation scheme (PTAS) and quasi-polynomial-time algorithms for Hub Labeling on trees; additionally, we analyze a simple combinatorial heuristic for Hub Labeling on trees, proposed by Peleg in 2000. We show that this heuristic gives an approximation factor of 2.