Abstract

The main result of this paper is the decidability of the membership problem for 2×2 nonsingular integer matrices. Namely, we will construct the first algorithm that for any nonsingular 2×2 integer matrices $M_1, \ldots, M_n$ and M decides whether M belongs to the semigroup generated by $\{M_1, \ldots, M_n\}$. Our algorithm relies on a translation of numerical problems on matrices into combinatorial problems on words. It also makes use of some algebraic properties of well-known subgroups of $\mathrm{GL}(2, \mathbb{Z})$ and various new techniques and constructions that help to convert matrix equations into the emptiness problem for intersection of regular languages.