Abstract

We consider the classic problem of scheduling jobs on unrelated machines so as to minimize the weighted sum of completion times. Recently, for a small constant $\varepsilon > 0$, Bansal et al. gave a $(3/2 - \varepsilon)$-approximation algorithm improving upon the "natural" barrier of $3/2$ which follows from independent randomized rounding. In simplified terms, their result is obtained by an enhancement of independent randomized rounding via strong negative correlation properties. In this work, we take a different approach and propose to use the same elegant rounding scheme for the weighted completion time objective as devised by Shmoys and Tardos for optimizing a linear function subject to makespan constraints. Our main result is a 1.21-approximation algorithm for the natural special case where the weight of a job is proportional to its processing time (specifically, all jobs have the same Smith ratio), which expresses the notion that each unit of work has the same weight. In addition, as a direct consequence of the rounding, our algorithm also achieves a bi-criteria 2-approximation for the makespan objective. Our technical contribution is a tight analysis of the expected cost of the solution compared to the one given by the Configuration-LP relaxation – we reduce this task to that of understanding certain worst-case instances which are simple to analyze.