Abstract

We consider the bin packing problem with d different item sizes and revisit the structure theorem given by Goemans and Rothvoß (*Polynomiality for Bin Packing with a Constant Number of Item Types*, SODA 2014) about solutions of the integer cone. We present new techniques on how solutions can be modified and give a new structure theorem that relies on the set of vertices of the underlying integer polytope. As a result of our new structure theorem, we obtain an algorithm for the bin packing problem with running time $|V|^{2^{O(d)}} enc(I)$, where V is the set of vertices of the integer knapsack polytope and $enc(I)$ is the encoding length of the bin packing instance. The algorithm is fixed parameter tractable, parameterized by the number of vertices of the integer knapsack polytope $|V|$. This shows that the bin packing problem can be solved efficiently when the underlying integer knapsack polytope has an easy structure, i.e. has a small number of vertices. Furthermore, we show that the presented bounds of the structure theorem are asymptotically tight. We give a construction of bin packing instances using new structural insights and classical number theoretical theorems which yield the desired lower bound.