Abstract

Most real world combinatorial optimization problems are affected by noise in the input data. Search algorithms to identify "good" solutions with low costs behave like the dynamics of large disordered particle systems, e.g., random networks or spin glasses. Such solutions to noise perturbed optimization problems are characterized by *Gibbs distributions* when the optimization algorithm searches for typical solutions by stochastically minimizing costs. The *free energy* that determines the normalization of the Gibbs distribution balances cost minimization relative to entropy maximization. The problem to analytically compute the free energy of disordered systems has been known as a notoriously difficult mathematical challenge for at least half a century (see M. Talagrand. *Spin Glasses: A Challenge for Mathematicians: Cavity and Mean Field Models.* Springer Verlag, 2003.). We provide rigorous, matching upper and lower bounds on the free energy for two disordered combinatorial optimization problems of random graph instances, the sparse Minimum Bisection Problem (sMBP) and Lawler's Quadratic Assignment Problem (LQAP). These two problems exhibit phase transitions that are equivalent to the statistical behavior of Derrida's Random Energy Model (REM). Both optimization problems can be characterized as *parameter rich* since individual solutions depend on more parameters than the logarithm of the solution space cardinality would suggest for e.g. a coordinate representation.