Abstract

Let $(X_1, \ldots, X_d)$ be a random nonnegative integer vector. Many conditions are known to imply a central limit theorem for a sequence of such random vectors, for example, independence and convergence of the normalized covariances, or various combinatorial conditions allowing the application of Stein's method, couplings, etc. Here, we prove a central limit theorem directly from hypotheses on the probability generating function $f(z_1, \ldots, z_d)$. In particular, we show that the f being *real stable* (meaning no zeros with all coordinates in the open upper half plane) is enough to imply a CLT under a nondegeneracy condition on the variance. Known classes of distributions with real stable generating polynomials include spanning tree measures, conditioned Bernoullis and counts for determinantal point processes. Soshnikov (2002) showed that occupation counts of disjoint sets by a determinantal point process satisfy a multivariate CLT. Our results extend Soshnikov's to the class of real stable laws. The class of real stable laws is much larger than the class of determinantal laws, being defined by inequalities rather than identities. Along the way we investigate the related problem of *stable multiplication*.